# 1 Properties of Steam

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## Definition of terms

### *Saturation Temperature*

Saturation temperature is the temperature at which liquids start to boil or the temperature at which vapors begin to condense. The saturation temperature of a given substance depends upon its existing pressure. It is directly proportional to the pressure, i.e., it increases as the pressure is increased and decreases as the pressure is decreased.

Examples:

a. Water boils at 100°C at atmospheric condition (101.325 kPa)
b. Water boils at 179.91°C at a pressure of 1000 kPa.
c. Steam condenses at 311.06°C at 10 MPa.
d. Steam condenses at 39°C at 0.0070 MPa.

### *Subcooled Liquid*

A subcooled liquid is one which has a temperature lower than the saturation temperature corresponding to the existing pressure.

Example:

Liquid water at 60°C and 101.325 is a subcooled liquid. Why?

From the steam tables, the saturation temperature at 101.325 kPa is 100°C. Since the actual temperature of liquid water of 60°C is less than 100°C, therefore, it is a subcooled liquid.

### *Compressed Liquid*

A compressed liquid is one which has a pressure higher than the saturation pressure corresponding to the existing temperature.

Question:

Is liquid water at 110 kPa and 100°C a compressed liquid?

From steam tables, $P_{sat}$ at 100°C = 101.325 kPa.

Comparing:

The actual liquid water pressure of 110 kPa is greater than $P_{sat}$ at 100°C. Therefore, it is a compressed liquid.

### Saturated Liquid

A saturated liquid is a liquid at the saturations (saturation temperature or saturation pressure) which has temperature equal to the boiling point corresponding to the existing pressure. It is a pure liquid, i.e., it has no vapor content.

Examples:

a. Liquid water at 100°C and 101.325 kPa.
b. Liquid water at 233.90°C and 3 MPa.
c. Liquid water at 324.75°C and 12 MPa.

From Steam Tables:

$t_{sat}$ at 101.325 kPa = 100°C
$t_{sat}$ at 3 MPa = 233.90°C
$t_{sat}$ at 12 MPa = 324.75°C

### Vapor

Vapor is the name given to a gaseous phase that is in contact with the liquid phase, or that is in the vicinity of a state where some of it might be condensed.

### Saturated Vapor

A saturated vapor is a vapor at the saturation conditions (saturation temperature and saturation pressure). It is 100% vapor, i.e., has no liquid or moisture content.

Examples:

a. Steam (water vapor) at 100°C and 101.325 kPa.
b. Steam at 212.42°C and 2 MPa.
c. Steam at 352.37°C and 17 MPa.

### Superheated Vapor

A superheated vapor is a vapor having a temperature higher than the saturation temperature corresponding to the existing pressure.

Examples:

a. Steam at 200°C and 101.325 kPa.
   200°C > ($t_{sat}$ at 101.325 kPa = 100°C)
b. Steam at 300°C and 5 MPa.
   300°C > ($t_{sat}$ at 5 MPa = 263.99°C)

### Degrees of Superheat, °SH

The degrees of superheat is the difference between the actual temperature of superheated vapor and the saturation temperature for the existing pressure.

In equation form:

°SH = Actual superheated temperature − $t_{sat}$ at existing pressure

Example:

Determine the degrees of superheat of superheated steam at 200°C and 101.325 kPa.

From steam tables:

$t_{sat}$ at 101.325 kPa = 100°C
°SH = 200 − 100 = 100 C°

### Degrees Subcooled, °SB

The degrees subcooled of a subcooled liquid is the difference between the saturation temperature for the given pressure and the actual subcooled liquid temperature.

°SB = $t_{sat}$ at given p − actual liquid temperature

**Example:**

Determine the degrees subcooled of liquid water at 90°C and 101.325 kPa.

From steam tables:

$t_{sat}$ at 101.325 kPa = 100°C

°SB = 100°C – 90°C = 10 C°

## *Wet Vapor*

A wet vapor is a combination of saturated vapor and saturated liquid.

## *Quality, x*

The quality of wet vapor or wet steam is the percent by weight that is saturated vapor.

## *Percent Moisture, y*

The percent moisture of wet vapor is the percent by weight that is saturated liquid.

Let $m$ = mass of wet vapor

$m_g$ = mass of the saturated vapor content of wet vapor

$m_f$ = mass of the saturated liquid content of wet vapor

Then, $m = m_g + m_f$

Following the definitions of quality (x) and percent moisture (y),

$$x = \frac{m_g}{m}(100)$$

$$y = \frac{m_f}{m}(100)$$

For saturated liquid:

$y = 100\%$

$x = 0\%$

For saturated vapor:

$x = 100\%$

$y = 0\%$

For wet vapor:

$0 < x < 100$

$0 < y < 100$

But $x + y = 100$ in percent form

$x + y = 1$ in decimal form

## *Latent Heat of Vaporization*

The latent heat of vaporization of a pure substance is the amount of heat added to/remove from the substance in order to convert it from saturated liquid/saturated vapor to saturated vapor/saturated liquid with the temperature remaining constant. It is inversely proportional to the temperature or pressure of the substance.

Example:

Determine the latent heat of vaporization of water at: (a) 100°C, (b) 200°C, and (c) 300°C.

From steam tables:

(a) $h_{fg}$ at 100°C = 2257.0 kJ/kg

(b) $h_{fg}$ at 200°C = 1940.7 kJ/kg

(c) $h_{fg}$ at 300°C = 1404.9 kJ/kg

## *Critical Point*

The critical point represents the highest pressure and highest temperature at which liquid and vapor can coexist in equilibrium. The state of water at critical conditions whether it is saturated liquid or saturated vapor is unknown. Hence, the latent heat of vaporization of water at this conditions is either zero or undefined.

## *Sensible Heat*

Heat that causes change in temperature without a change in phase.

Examples:

a. Heat added in raising the temperature of steam from 100°C at 101.325 kPa to 150°C.

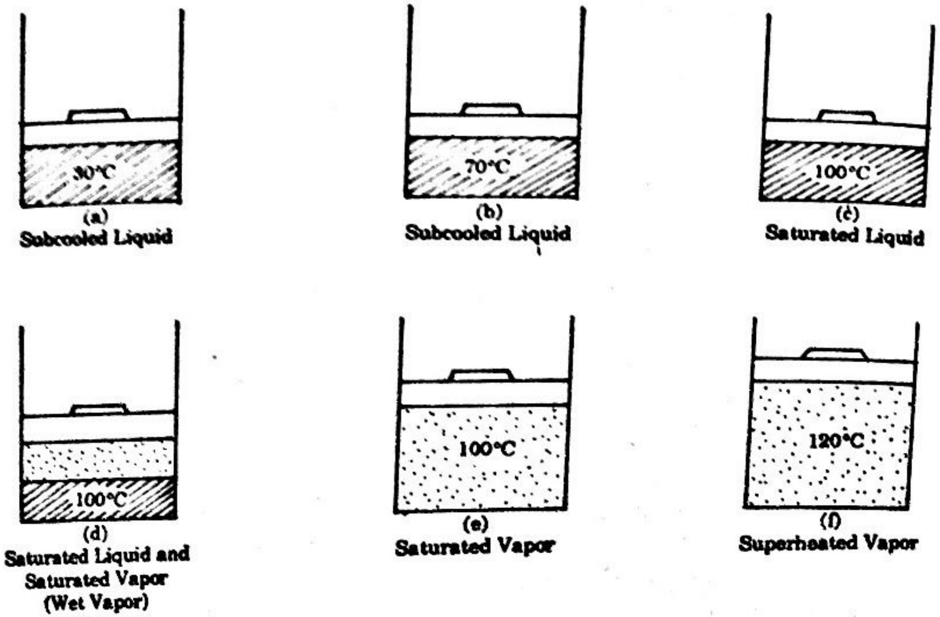
b. Heat removed in lowering the temperature of water from 90°C to 80°C.

***Latent Heat***

Heat that causes change in phase without a change in temperature.

**Example:**
Heat added in converting 1 kg of water at 100°C and 101.325 kPa to 1 kg of steam at 100°C and 101.315 kPa.

## Change of Phase at Constant Pressure


30°C
70°C
100°C
100°C
100°C
120°C

(a) Subcooled Liquid
(b) Subcooled Liquid
(c) Saturated Liquid
(d) Saturated Liquid and Saturated Vapor (Wet Vapor)
(e) Saturated Vapor
(f) Superheated Vapor

**Fig. 1-1. Heating at Constant Pressure.**

Let the piston-cylinder arrangement contains *subcooled liquid* water at 30°C and 101.325 kPa (Fig. a). Then let heat be added continuously at constant pressure. First, the water gets warmer. The addition of (*sensible*) heat causes an increased in water temperature, say, from 30°C to 70°C (Fig. b). Further addition of (sensible) heat causes a further increased in water temperature until it reaches 100°C, the *boiling or saturation temperature* at 101.325 kPa. Water at this point is at *saturated liquid* state (Fig. c). Then as more (*latent*) heat is added to the now *saturated liquid water* causes some of it to be converted to saturated vapor with the temperature remaining at 100°C. Water is now a mixture of saturated liquid and saturated vapor. Hence, it is now in *wet vapor state* (Fig. d). Further addition of (latent) heat causes the transformation of the remaining saturated liquid water to saturated water vapor at 100°C. Water is now at the *saturated vapor state* (Fig. e). Finally, as more (sensible) heat is added to the saturated water vapor at 100°C its temperature rises say until it reaches 110°C. Water is now at the *superheated vapor* state (Fig. f).

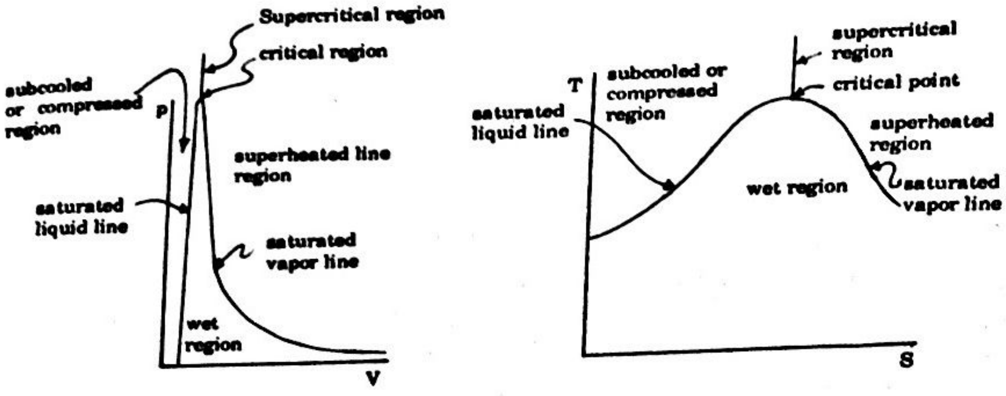

Supercritical region
critical region
subcooled or compressed region
P
superheated line region
saturated liquid line
saturated vapor line
wet region
V
T
subcooled or compressed region
saturated liquid line
supercritical region
critical point
superheated region
wet region
saturated vapor line
S

**Fig. 1-2. The pv and Ts Planes for Water**

## Properties of Wet Steam

Subscripts Used/Symbols Used.

f – represents properties of saturated liquid
g – represents properties of saturated vapor
fg – represents to a change by evaporation
v – specific volume, m³/kg
s – specific entropy, kJ/kg.K
u – specific internal energy, kJ/kg

$$\begin{array}{llll} v_g = v_f + v_{fg} & & & v_{fg} = v_g - v_f \\ h_g = h_f + h_{fg} & & & h_{fg} = h_g - h_f \\ s_g = s_f + s_{fg} & & & s_{fg} = s_g - s_f \\ u_g = u_f + u_{fg} & & & u_{fg} = u_g - u_f \end{array}$$

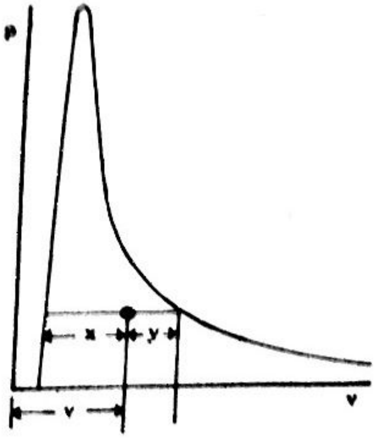


**Fig. 1-3. Specific Volume of a Wet Mixture.**

v = specific volume of its saturated liquid content + specific volume of its saturated vapor content

$$v = yv_f + xv_g$$

$$yv_f = \frac{\text{kg liquid}}{\text{kg mixture}} \bullet \frac{m^3}{\text{kg liquid}} = \frac{m^3}{\text{kg mixture}}$$

$$xv_g = \frac{\text{kg vapor}}{\text{kg mixture}} \bullet \frac{m^3}{\text{kg vapor}} = \frac{m^3}{\text{kg mixture}}$$

From $v = yv_f + xv_g$

But $y = 1 - x$

Then $v = (1 - x)v_g + xv_g$

$= v_f + x(v_g - v_f)$

$v = v_f + xv_{fg}$

or $x = 1 - y$

$v = yv_f + (1 - y)v_g$

$= v_g - y(v_g - v_f)$

$v = v_g - yv_{fg}$

Similarly,

$h = h_f + xh_{fg}$ or $h = hg - yh_{fg}$

$s = s_f + xs_{fg}$ or $s = s_g - ys_{fg}$

$u = u_f + xu_{fg}$ or $u = u_g - yu_{fg}$

**NOTE:**

At saturated conditions, p and t are dependent on one another and therefore, are considered as one independent property. At superheated conditions, p and t are independent from each other and therefore, are considered as two independent properties.

Quality could not be more than 100% and per cent moisture could not be lower than 0%.

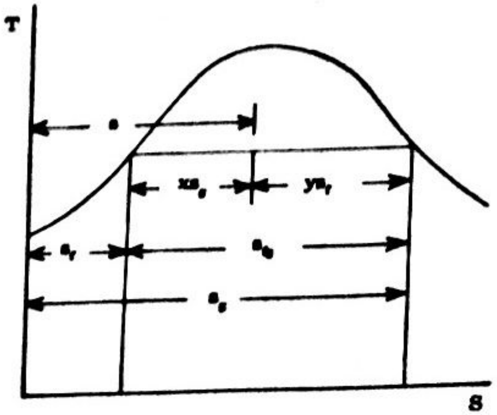


**Fig. 1-4. Specific Entropy of a Wet Mixture.**

(1–1) Specify whether the steam is wet, dry, or superheated for the following conditions:

(a) t = 200°C, p = 1.44 MPa
(b) t = 220°C, p = 2.318 MPa.
(c) p = 1.0 MPa, s = 6.672 kJ/kg•K
(d) p = 3.0 MPa, t = 234°C
(e) t = 250°C, v = $54.2 \times 10^{-3}$ m³/kg
(f) p = 11.0 MPa, h = 2805 kJ/kg
(g) p = 4.0 MPa, s = 5.897 kJ/kg•K
(h) p = 15.0 MPa, t = 310°C

**SOLUTION:**

(a) $t_{sat}$ at 1.44 MPa = 196.39°C
Since 200°C > $t_{sat}$ at 1.44 MPa, the steam is superheated.

(b) $t_{sat}$ at 2.318 MPa = 220°C
Since 220°C = $t_{sat}$ at 2.318 MPa, the steam is dry saturated.

(c) $s_g$ at 1.0 MPa = 6.5865 kJ/kg•K
Since 6.672 kJ/kg•K > $s_g$ at 1.0 MPa, the steam is superheated.

(d) $t_{sat}$ at 3.0 MPa = 233.90°C
Since 234°C > $t_{sat}$ at 3.0 MPa, the steam is superheated.

(e) $v_g$ at t = 250°C = 50.13x10$^{-3}$ m$^3$/kg
Since v = 54.2x10$^{-3}$/kg > $v_g$ at 250°C, the steam is superheated.

(g) $s_g$ at 4.0 MPa = 6.0701 kJ/kg•K
Since s = 5.897 kJ/kg• K< $s_g$ at 4.0 MPa, the steam is a wet mixture.

(h) $t_{sat}$ at 15.0 MPa = 342.24°C
Since t = 310°C < $t_{sat}$ at 15.0 MPa, the steam is a wet mixture.

(1-2) What are the specific volume, internal energy, enthalpy, and entropy of steam at 1.50 MPa and 254°C?

**SOLUTION**

The steam is superheated, since the temperature of 254°C is greater than the saturation temperature, 198°C, at 1.50 MPa.

At 1.50 MPa

| | | | | |
|---|---|---|---|---|
| 250 | 0.15195 | 2695.3 | 2923.3 | 6.7090 |
| 254 | v | u | h | s |
| 260 | 0.15559 | 2713.4 | 2946.7 | 6.7535 |
| 4 | x1 | x2 | x3 | x4 |
| 10 | 0.00364 | 18.1 | 23.4 | 0.0445 |

$$\frac{4}{10} = \frac{x_1}{0.00364} = \frac{x_2}{18.1} = \frac{x_3}{23.4} = \frac{x_4}{0.0445}$$

$x_1$ = 0.00146; v = 0.15195 + 0.00146 = 0.15341 m$^3$/kg

$x_2$ = 7.24; u = 2695.3 + 7.24 = 2702.5 kJ/kg

$x_3$ = 9.36; h = 2923.3 + 9.36 = 2932.7 kJ/kg

$x_4$ = 0.0178; s = 6.7090 + 0.0178 = 6.7268 kJ/(kg) (K)

(1-3) At 250°C a mixture of saturated steam and liquid water exists in equilibrium. If the specific volume of the mixture is 0.04159 m$^3$/kg, calculate the following: (a) percent moisture, (b) enthalpy, and (c) entropy.

**SOLUTION:**

At 250°C

$v_f$ = 0.0012512 $h_f$ = 1085.36 $s_f$ = 2.7927
$v_{fg}$ = 0.04888 $h_{fg}$ = 1716.2 $s_{fg}$ = 3.2802
$v_g$ = 0.05013 $h_g$ = 2801.5 $s_g$ = 6.0729

$$v = v_f + xv_{fg}$$

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.04159 - 0.0012512}{0.04888} = 0.8253 \text{ or } 82.53\%$$

(a) y = 100 – 82.53 = 17.47%
(b) h = $h_f + xh_{fg}$
h = 1085.36 + (0.8253) (1716.2) = 2501.7 kJ/kg
(c) s = $s_f + xs_{fg}$
s = 2.7927 + (0.8253) (3.2802) = 5.4998 kJ/(kg) (K)

(1-4) Steam at a pressure of 0.90 MPa has an entropy of 4.9678 kJ/(kg) (K). What are the specific volume, internal energy, and enthalpy?

**SOLUTION:**

The entropy of saturated vapor at a pressure of 0.90 MPa is 6.6185 kJ/(kg) (K), therefore, steam with an entropy of 4.9678 kJ/(kg) (K) is a wet-mixture.

At 0.90 MPa

$v_f$ = 0.0011212 $u_f$ - 741.83 $h_f$ = 742.83 $s_f$ = 2.0946
$v_{fg}$ = 0.21388 $u_{fg}$ = 1838.6 $h_{fg}$ = 2031.1 $s_{fg}$ = 4.5280
$v_g$ = 0.215 $u_g$ = 2580.5 $h_g$ = 2773.91 $s_g$ = 6.6226

$$s = s_f + x s_{fg}$$

$$x = \frac{s - s_f}{s_{fg}} = \frac{4.9678 - 2.0946}{4.5280} = 0.6345$$

$$v = v_f + x v_{fg}$$

$$v = 0.0011212 + (0.6345)(0.21388) = 0.1368 \text{ m}^3/\text{kg}$$

$$u = u_f + x u_{fg}$$

$$u = 741.83 + 0.6345\,(1838.6) = 1908.4\ \frac{\text{kJ}}{\text{kg}}$$

$$h = h_f + x h_{fg}$$

$$h = 742.83 + (0.6345)(2031.1) = 2031.6 \text{ kJ/kg}$$

(1-5) Steam at a temperature of 300°C has a specific volume of 0.09765 m³/kg. Determine the pressure, specific internal energy, enthalpy, and entropy.

**SOLUTION:**

Since the specific volume, 0.09765 m³/kg, is greater than the specific volume of saturated steam, 0.02167 m³/kg, at 300°C, the steam is superheated.

At 300°C

$$\begin{array}{ccl|cc|c|cc|c|cc|c|cc|c}
 & & 0.09681 & 2.55 & & & 2760.4 & & & 3007.3 & & & 6.6326 & & \\
0.00209 & 0.00084 & 0.09765 & p & x1 & 0.05 & u & x2 & 1.20 & h & x3 & 1.5 & s & x4 & 0.0112 \\
 & & 0.09890 & 2.50 & & & 2761.6 & & & 3008.8 & & & 6.6438 & &
\end{array}$$

$$\frac{0.00084}{0.00209} = \frac{x_1}{0.05} = \frac{x_2}{1.2} = \frac{x_3}{1.5} = \frac{x_4}{0.0112}$$

$x_1$ = 0.02; $p = 2.55 - 0.02 = 2.53$ kPa

$x_2$ = 0.50; $u = 2760.4 + 0.5 = 2760.9$ kJ/kg

$x_3$ = 0.60; $h = 3007.3 + 0.6 = 3007.9$ kJ/kg

$x_4$ = 0.0045; $s = 6.6326 + 0.0045 = 6.6371$ kJ/(kg) (K)

**(1-6) Steam at a pressure of 0.58 MPa has an internal energy of 2725.2 kJ/kg. What are the temperature, specific volume, enthalpy, and entropy?**

**SOLUTION:**

**The steam is superheated because the internal energy, 2725.2 kJ/kg, is greater than the internal energy of saturated steam, 2566.3 kJ/kg, for a pressure of 0.58 MPa.**

**At 0.58 MPa**

$$\begin{array}{ccl|cc|c|cc|c|cc|c|cc|c}
 & & 250. & 0.4077 & & & 2721.4 & & & 2957.9 & & & 7.1983 & & \\
10 & x1 & t & v & x2 & 0.0085 & 2725.2 & 3.8 & 16.1 & h & x3 & 20.9 & s & x4 & 0.0397 \\
 & & 260 & 0.4162 & & & 2737.5 & & & 2978.8 & & & 7.2380 & &
\end{array}$$

$$\frac{3.8}{16.1} = \frac{x_1}{10} = \frac{x_2}{0.0085} = \frac{x_3}{20.9} = \frac{x_4}{0.0397}$$

$x_1$ = 2.4; $t = 250 + 2.4 = 252.4$°C

$x_2$ = 0.002; $v = 0.4077 + 0.002 = 0.4097$ m³/kg

$x_3$ = 4.9; $h = 2957.9 + 4.9 = 2962.8$ kJ/kg

$x_4$ = 0.0094; $s = 7.1983 + 0.0094 = 7.2077$ kJ/(kg)(K)

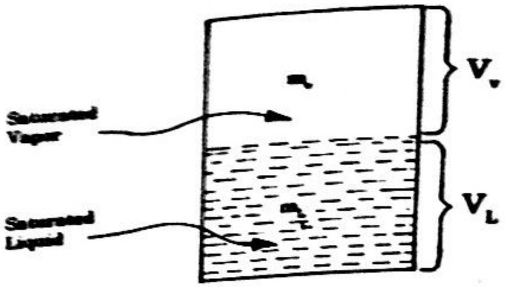
(1-7) A 0.0856-m³ drum contains saturated water and saturated vapor at 370°C. (a) Find the mass of each if their volumes are equal. What is the quality? (b) Find the volume occupied by each if their masses are equal.

**SOLUTION:**

Let $V_v$ = the volume of saturated vapor
$V_L$ = the volume of saturated liquid
$m_v$ = the mass of saturated vapor
$m_L$ = the mass of saturated liquid

$v_g$ at 370°C = 0.004925 m³/kg
$v_f$ at 370°C = 0.002213 m³/kg

(a)


Saturated
Vapor
Saturated
Liquid
$m_v$
$m_L$
$V_v$
$V_L$

$$V_v = V_L = \frac{0.0856}{2} = 0.0428\ m^3$$

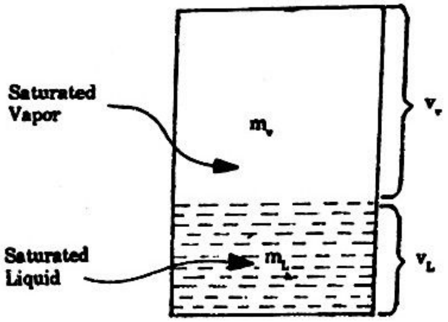
$$m_v = \frac{V_v}{v_g} = \frac{0.0428}{0.004925} = 8.69\ kg$$

$$m_L = \frac{V_L}{v_f} = \frac{0.0428}{0.002213} = 19.34\ kg$$

$$\text{quality, } x = \frac{\text{mass of the vapor}}{\text{mass of the mixture}}$$

$$x = \frac{8.69}{8.69 + 19.34} = 0.31 \text{ or } 31\%$$

(b)


Saturated
Vapor
Saturated
Liquid
$m_v$
$m_L$
$V_v$
$V_L$

$m_v = m_L$ (1)

$V_v + V_L = 0.0856$ (2)

$V_v = m_v v_g = 0.004925 m_v$

$V_L = m_L v_f = 0.002213 m_L$

substituting in equation (2),

$$0.004925 m_v + 0.002213 m_L = 0.0856$$

$$0.004925 m_v + 0.002213 m_v = 0.0856$$

$$m_v = 11.99\ kg$$

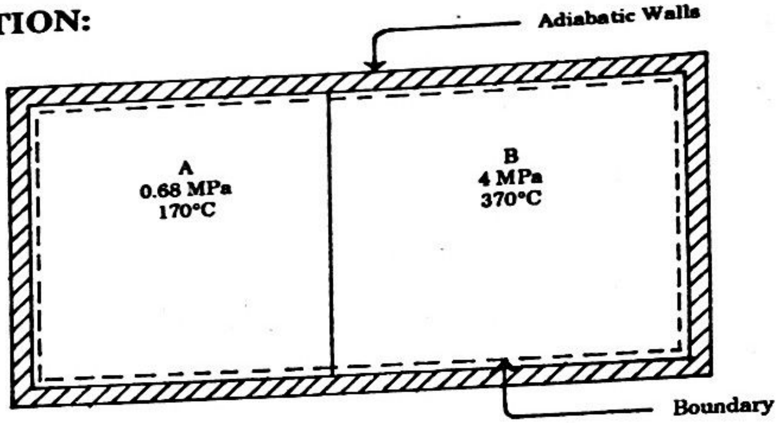
$$m_L = 11.99\ kg$$

$$V_v = m_v v_g = (11.99)(0.004925) = 0.05905\ m^3$$

$$V_L = m_L v_f = (11.99)(0.002213) = 0.02653\ m^3$$

(1-8) A 60-liter rigid tank with adiabatic walls is divided into equal parts A and B by a partition. On one side is steam at 0.68 MPa, 170°C; on the other side is steam at 4 MPa, 370°C. The partition is removed and thorough mixing occurs. Determine the equilibrium state (p, t) neglecting the thickness of the partition.

## SOLUTION:


Adiabatic Walls
A
0.68 MPa
170°C
B
4 MPa
370°C
Boundary

System: Steam

From Steam Tables:

$h_A$ = h at 0.68 MPa and 170°C = 2777 kJ/kg

$u_A$ = u at 0.68 MPa and 170°C = 2582.9 kJ/kg

$v_A$ = v at 0.68 MPa and 170°C = $285.5 \times 10^{-3}\ m^3/kg$

$h_B$ = h at 4MPa and 370°C = 3141.7 kJ/kg

$u_B$ = h at 4 MPa and 370°C = 2864.6 kJ/kg

$v_B$ = h at 4 MPa and 370°C = $69.29 \times 10^{-3}\ m^3/kg$

**Law of conservation of mass:**

Initial stored mass = Final stored mass

$$m_A + m_B = m_C$$

$$m_A = \frac{V_A}{v_A} = \frac{0.030}{0.2855} = 0.105 \text{ kg}$$

$$m_B = \frac{V_B}{v_B} = \frac{0.030}{0.06929} = 0.433 \text{ kg}$$

Therefore, $m_C = 0.105 + 0.433 = 0.538$ kg

$$V_C = V_A + V_B = 60 \text{ L} = 0.060 \text{ m}^3$$

Specific volume, $v_C = \frac{V_c}{m_c} = \frac{0.60}{0.538} = 0.1115 \text{ m}^3/\text{kg}$

or $111.5 \times 10^{-3}$ $\text{m}^3/\text{kg}$

**First law of Thermodynamics:**

Initial stored energy = Final stored energy

$$U_A + U_B = U_C$$

$$m_A u_A + m_B u_B = m_C u_C$$

$$u_C = \frac{m_A u_A + m_B u_B}{m_C}$$

$$= \frac{(0.105)(2582.9) + (0.433)(2864.6)}{0.538}$$

$$= 2809.62 \text{ kJ/kg}$$

Heat balance:

Heat lost by Steam B = Heat gained by Steam A

$$m_B c\,(t_B - t_C) = m_A c\,(t_C - t_A)$$

$$m_B c t_B + m_A c t_A = (m_A + m_B)\, c t_C$$

$$m_B h_B + m_A h_A = m_C h_C$$

$$h_C = \frac{m_B h_B + m_A h_A}{m_C}$$

$$= \frac{(0.433)(3141.7) + (0.105)(2777)}{0.538}$$

$$= 3070.52 \text{ kJ/kg}$$

By definition, $h_C = u_C + p_C v_C$

$$p_C = \frac{h_C - u_C}{v_C} = \frac{3070.52 - 2809.62}{0.1115}$$

$$= 2339.9 \text{ kPa}$$

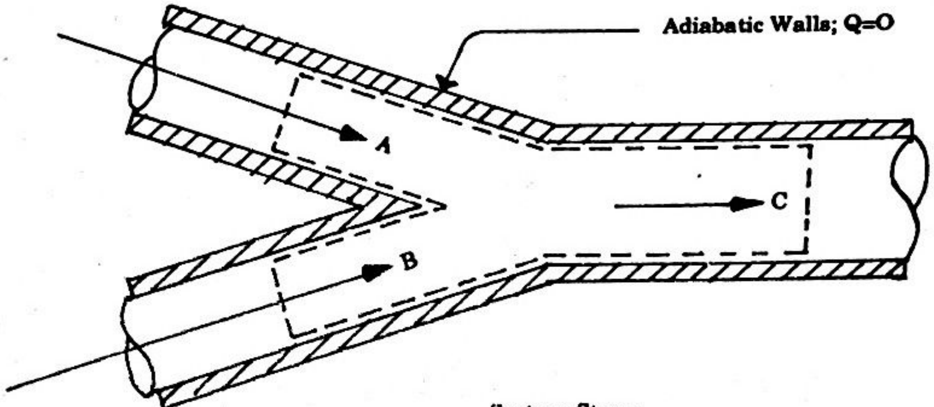
$$= 2.34 \text{ MPa}$$

**From the Mollier Chart**

**t = (t at 3070.52 kJ/kg and 2.34 MPa) = 324°C**

**(1.9) Steadily flowing streams of steam A and B meet and mix, then flow in a steady state in a pipe C, all under adiabatic conditions. Stream A is at 5.5 MPa, 370°C and flows at 45 kg/s; stream B is saturated vapor at 5.5 MPa and flows at 160 kg/s. (a) Determine the state of the mixture C if $\Delta KE \approx 0$ and $\Delta PE \approx 0$. (b) If the speed of the fluid in C is 30 m/s, what is the pipe diameter?**

**SOLUTION:**



System: Steam

(a) From Steam Tables:

Steam A:

$h_A = h$ at 5.5 MPa and 370°C = 3109.6 kJ/kg

Steam B:

$h_B = h_g$ at 5.5 MPa = 2789.65 kJ/kg (by interpolation)

Since $p_A = p_B = 5.5$ MPa

Then if follows that $p_C = 5.5$ MPa

Law of conservation of mass:

$$m_{in} = m_{out}$$

$$m_A + m_B = m_C$$

$$m_C = 45 + 160 = 205 \text{ kg/s}$$

First law of Thermodynamics:

$$E_{in} = E_{out}$$

$$E_A + E_B = E_C$$

$$U_A + p_A V_A + U_B + p_B V_B = p_C V_C + U_C$$

$$H_A + H_B = H_C$$

$$m_A h_A + m_B h_B = m_C$$

$$hc = \frac{m_A h_A + m_B h_B}{m_C}$$

$$= \frac{(45)(3109.6) + (160)(2789.65)}{205}$$

$$= 2859.88 \text{ kJ/kg}$$

To determine the state of C compare $h_C$ with hg at $p_C = 5.5$ MPa.

$$[h_g \text{ at } p_C = 5.5 \text{ MPa}] = 2789.65 \text{ kJ/kg}$$

Since $h_C > h_g$ at $p_C$, therefore, the state of the mixture is superheated.

From the Superheat Tables

5.5 MPa

| t | h | v |
|---|---|---|
| 285 | 2851.5 | $38.13 \times 10^{-3}$ |
| $t_c$ | 2859.88 | $v_c$ |
| 290 | 2869.8 | $38.89 \times 10^{-3}$ |

(b) By interpolations:

$$t_c = 287.34°\text{C}$$
$$v_c = 38.49 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$m_C = \frac{A_C \upsilon C}{v_C}$$

$$A_C = \frac{m_C v_C}{\upsilon_c}$$

$$= \frac{(205)(0.003849)}{30}$$

$$= 0.02630 \text{ m}^2$$

$$A_C = \frac{\pi}{4} D^2{}_C$$

$$D_C = \sqrt{\frac{4A_C}{\pi}}$$

$$= \sqrt{\frac{(4)(0.02630)}{\pi}}$$

$$= 0.183 \text{ m or } 183 \text{ mm}$$

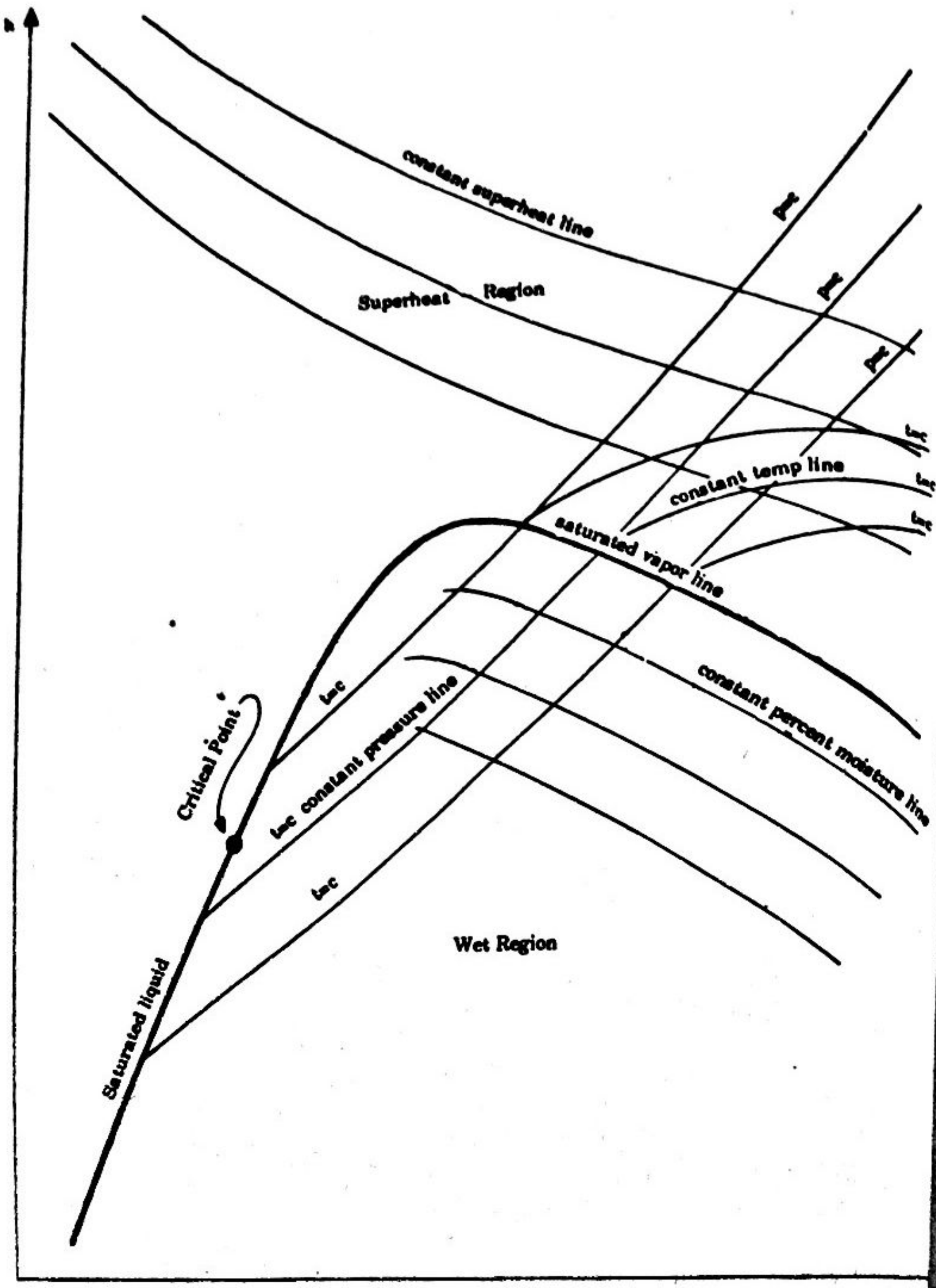

h
constant superheat line
Superheat Region
p=c
p=c
p=c
t=c
t=c
t=c
constant temp line
saturated vapor line
constant percent moisture line
t=c constant pressure line
t=c
t=c
Critical Point
Saturated liquid
Wet Region

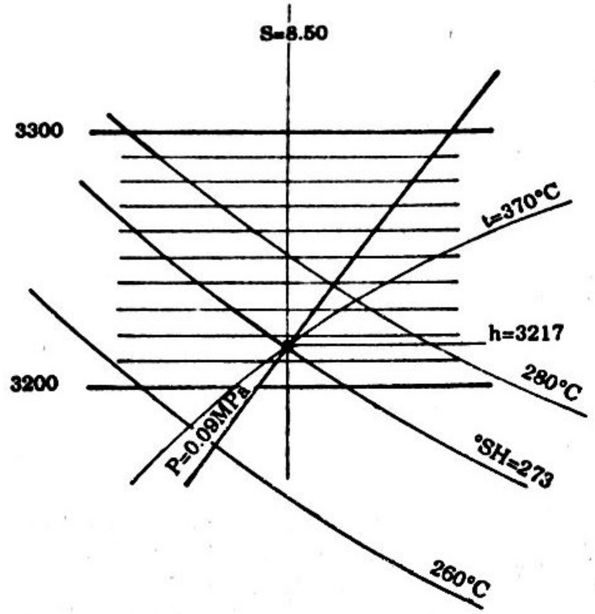
Fig. 1-5. Enthalpy-Entropy (Mollier) Diagram for Steam.

The enthalpy-entropy chart (h-s chart), generally known as a Mollier diagram, is a chart on which enthalpy is the ordinate and entropy the abscissa. This chart can be found at the back of Steam Tables by Keenan, Keyes, Hill and Moore. It is presented in skeleton form on Fig. 1.5. On this chart, a series of constant pressure lines, a series of constant percent moisture and superheat lines, and a series of constant temperature lines are plotted. The constant temperature lines coincide with the constant pressure lines in the wet region (below the saturated vapor line) but bend toward the right away from the constant pressure lines in the superheat region.

**(1-10) Determine the h, s, °SH of steam at 0.09 MPa and 370°C.**

## SOLUTION:

Locate the intersection of 0.09 MPa constant pressure line with 370°C constant temperature line. Their intersection represents the condition/state of steam.


S=8.50
3300
3200
t=370°C
h=3217
280°C
°SH=273
260°C
P=0.09MPa

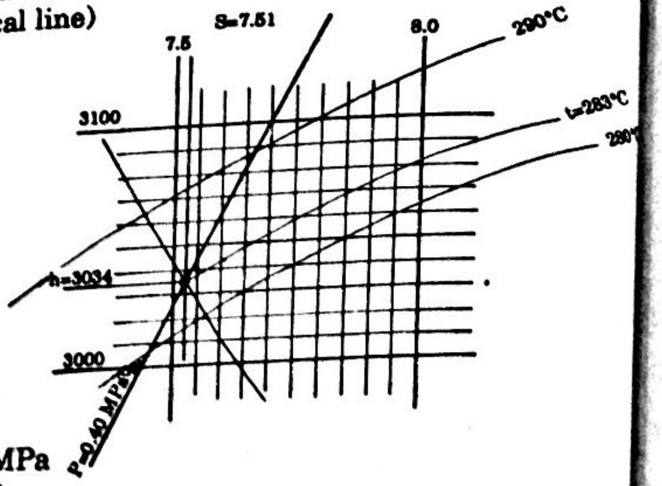
Answers:

h = 3217 kJ/kg
s = 8.50 kJ/kg•K°
°SH = 273°C

(1-11) Determine the pressure and temperature of steam having h = 3034 kJ/kg and s = 7.51 kJ/kg•K°.

**SOLUTION:**

Locate the intersection of h = 3034 kJ/kg constant enthalpy line (horizontal line) and s = 7.51 kJ/kg•K° constant entropy line (vertical line)


S=7.51
7.5
8.0
290°C
3100
t=283°C
280°
h=3034
3000
P=0.40 MPa

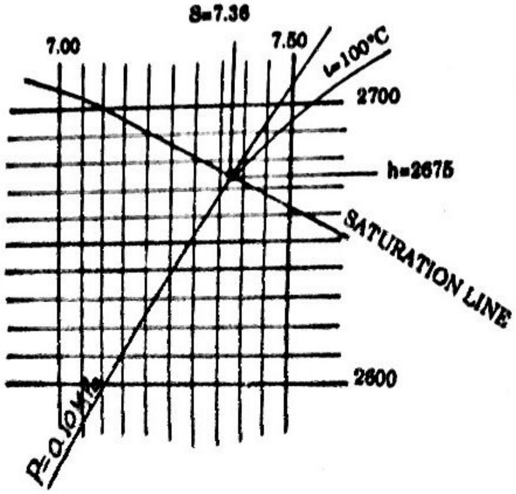
Answers:

p = 0.40 MPa
t = 283°C

(1-12) Determine the h, t, and s of saturated steam at 0.10 MPa.

**SOLUTION:**

Locate the intersection of 0.10 MPa constant pressure line with the saturated vapor line.

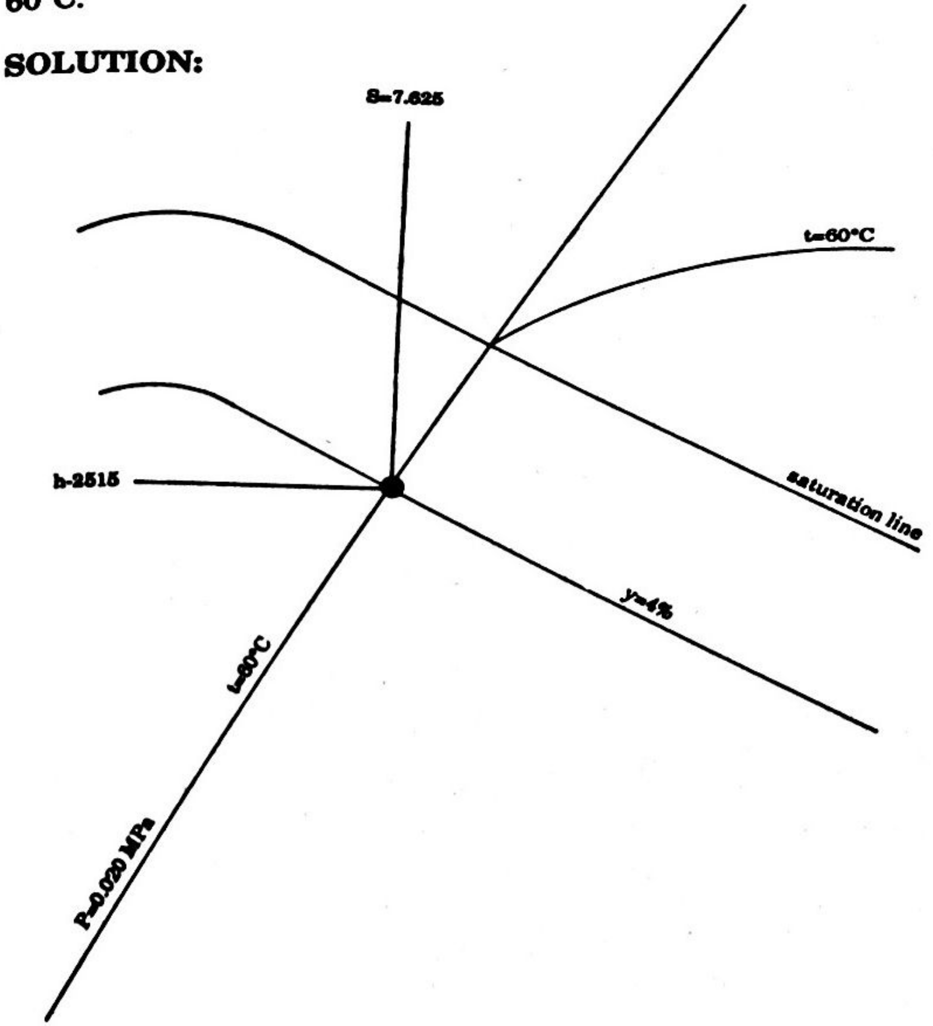

S=7.36
7.00
7.50
t=100°C
2700
h=2675
SATURATION LINE
2600
P=0.10 MPa

**Answers:**

h = 2675 kJ/kg
t = 100°C
s = 7.36 kJ/kg•K°

(1-13) Determine the p and h of steam at y = 4% and t = 60°C.

**SOLUTION:**


S=7.625
t=60°C
h=2515
saturation line
y=4%
t=60°C
P=0.020 MPa

The isothermal lines and isobaric lines coincide in the wet region.

Answers:

h = 2515 kJ/kg
p = 0.020 MPa

## Problems

(1) Steam at a temperature of 250°C has a specific volume of 0.15436m$^3$/kg. What are the pressure, specific internal energy, enthalpy, and entropy? Ans. 1.48 MPa, 2696 kJ/kg, 2924.2 kJ/kg, 6.717 kJ/kg•K

(2) Steam at a pressure of 3.5 MPa is known to have a specific volume of 50x10$^{-3}$ m$^3$/kg. What is its specific enthalpy? Ans. 2581.39 kJ/kg

(3) A rigid vessel contains 2 kg of steam at 6.35 MPa and 375°C. Determine its enthalpy in kJ using steam tables only. Ans. 6208.15 kJ

(4) A tank contains exactly one kilogram of water consisting of liquid and vapor in equilibrium at 1 MPa. If the liquid and vapor each occupy one-half the volume of the tank, what is the enthalpy of the contents of the tank? Ans. 774.4 kJ/kg

(5) Steam at a temperature of 210°C has a specific entropy of 7.1167 kJ/kg•K. Determine the pressure, internal energy, enthalpy and volume. Ans. 0.49 MPa, 2877.3 kJ/kg, 2659.7 kJ/kg, 0.4467 m$^3$/kg

(6) Saturated steam vapor at 250°C moves along its isotherm until the pressure becomes 1 MPa. Locate the end state points on the Mollier chart and for each kilogram of steam processed determine (a) the initial pressure, (b) the change of enthalpy, and (c) the change of entropy. Ans. (a) 4 MPa, (b) 144 kJ/kg, (c) 0.90 kJ/kg•K

(7) Steam undergoes an isentalpic (h = C) process from 1.5 MPa, 350°C to 10 MPa. Find the final temperature using the Mollier chart. Ans. 418°C

(8) Consider 1 kg of water that is heated at constant pressure (0.50 MPa) from a wet state ($x_1$ = 85%) to 400°C. Locate the end state points on the Mollier chart and determine (a) the change of enthalpy and (b) the change of entropy.

Ans. (a) 739 kJ/kg, (b) 1.70 kJ/kg

# 2 Processes of Vapors

## Introduction

Although vapors and ideal gases have similarity in forms and in their processes, all of the equations that are based on the characteristic equation of a perfect gas or on Joule's Law, for instance, are not generally applicable to vapors. But the general energy equation,

$$P_1 + K_1 + U_1 + W_{f1} + Q = P_2 + U_2 + W_{f2} + W$$

and the simple energy equation,

$$Q = U_2 - U_1 + W_n$$

are not based on any limitations concerning the substance. They are applicable, therefore, to processes of vapors as well as of gases. Therefore in this chapter, compare the equations obtained with analogous one for a gas. This way we can avoid the improper application of a perfect gas equation to a vapor.

## Constant Pressure Process

(a) The process on the pv and Ts planes.

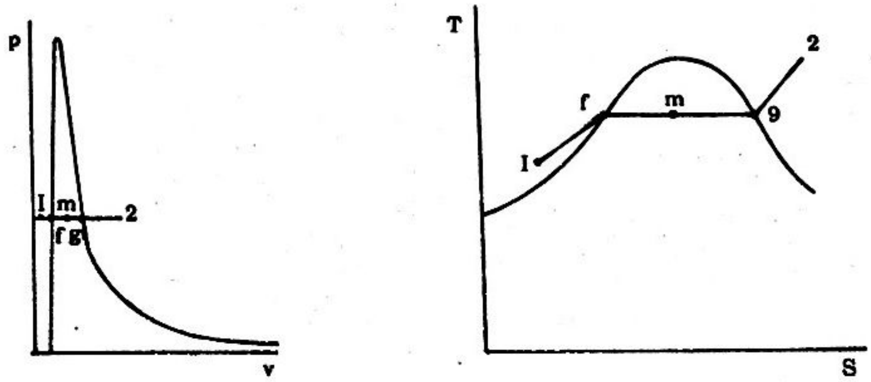


**Fig. 2-1. Constant Pressure Process.**

### Defining the condition of the substance

Point 1 is in the liquid region and the condition of the substance is "subcooled" or "compressed" liquid. It is generally defined by giving its pressure and temperature.

Point f lies on the saturation curve and is therefore saturated liquid at a given pressure or temperature.

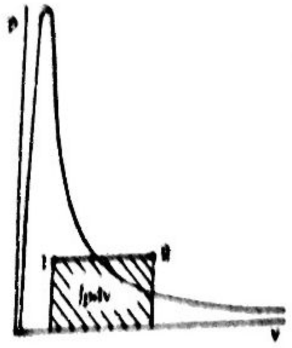
Point m is in the wet region, is a mixture of liquid and vapor. A quality x and a pressure or temperature generally defines the condition of the substance. Thus, for the enthalpy at point m,

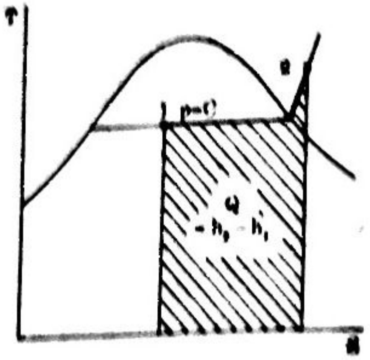
$$h_m = h_{fm} + xh_{fgm}$$

Point g is on the saturation vapor curve. A pressure or temperature defines the state or condition of the substance.

Point 2 in the superheat region is generally, but not necessarily, defined by giving its temperature and pressure.

(b) The work of a reversible nonflow constant pressure process.





$W_n = \int pdv$ is the area on the pv plane under the constant pressure process.

At p = C,

$W_n = p \int dv = p(v_2 - v_1)$ [unit mass]

For steady flow process, the work $W_s$ is

$$W_s = h_1 - h_2 - \Delta K + Q$$

and $h_1 = h_{f1} + x_1h_{fg1}$

(e) Transferred heat, Q.

From the simple energy equation,

$Q = u_2 - u_1 + W_n$ [unit mass]

For steady flow and nonflow processes,

$$\begin{aligned} Q &= u_2 - u_1 + p(v_2 - v_1) \\ &= u_2 - u_1 + p_2v_2 - p_1v_1 \\ &= h_2 - h_1 \end{aligned}$$

where $h_1 = h_{f1} + x_1h_{fg1}$

If the change in specific internal energy is desired, we use the relation,

$$u_2 - u_1 = (h_2 - p_2v_2) - (h_1 - p_1v_1)$$

(2-1) Steam with a specific volume of 0.09596 m³/kg undergoes a constant pressure process at 1.70 MPa until the specific volume becomes 0.13796 m³/kg. What are (a) the final temperature, (b) Δu, (c) W, (d) Δs, and (e) Q?

**SOLUTION:**

Steam at a pressure of 1.70 MPa and with a specific volume of 0.09596 m³/kg is a wet-mixture, the specific volume of saturated steam at 1.70 MPa is 0.11673 m³/kg.

At $p_1$ = 1.70 MPa

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $v_{f1}$ = | 0.0011634 | $u_{f1}$ | = 870.09 | $h_{f1}$ | = 872.06 | $s_{f1}$ | = 2.3718 |
| $v_{fg1}$ = | 0.11557 | $u_{fg1}$ | = 1727.2 | $h_{fg1}$ | = 1923.6 | $s_{fg1}$ | = 4.0282 |
| $v_{g1}$ = | 0.11673 | $u_{g1}$ | = 2597.3 | $h_{g1}$ | = 2795.7 | $s_{g1}$ | = 6.400 |

$$x_1 = \frac{v_1 - v_{f1}}{v_{fg1}} = \frac{0.09596 - 0.0011634}{0.11557} = 0.8203$$

$u_1 = u_{f1} + x_1 u_{fg1} = 870.09 + (0.8203)(1727.2) = 2286.9$ kJ/kg

$h_1 = h_{f1} + x_1 h_{fg1} = 872.06 + (0.8203)(1923.6) = 2450.0$ kJ/kg

$s_1 = s_{f1} + x_1 s_{fg1} = 2.3718 + (0.8203)(4.0282) = 5.6761$ kJ/(kg)(K)

At a pressure of 1.70 MPa and a specific volume of 0.13796 m³/kg, the steam is superheated

At $p_2 = 1.70$ MPa

$$0.00323\left[0.00175\left[\begin{array}{cc} 0.13621 & 260 \\ 0.13796 & t_2 \\ 0.13944 & 270 \end{array}\right] x_1\right] 10 \left[\begin{array}{c} 2707.7 \\ u_2 \\ 2725.9 \end{array}\right] x_2 \Bigg] 18.2 \left[\begin{array}{c} 2939.3 \\ h_2 \\ 2963.0 \end{array}\right] x_3 \Bigg] 23.7 \left[\begin{array}{c} 6.6849 \\ s_2 \\ 6.7290 \end{array}\right] x_4 \Bigg] 0.0441$$

$$\frac{0.00175}{0.00323} = \frac{x_1}{10} = \frac{x_3}{18.2} = \frac{x_4}{0.0441}$$

$x_1 = 0.542;\ t_2 = 260 + 5.42 = 265.4°C$

$x_2 = 9.86;\ u_2 = 2707.7 + 9.86 = 2717.6$ kJ/kg

$x_3 = 12.84;\ h_2 = 2939.3 + 12.84 = 2952.1$ kJ/kh

$x_4 = 0.02389;\ s_2 = 6.6849 + 0.02389 = 6.7088$ kJ/(kg) (K)

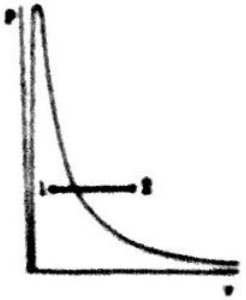
(a) $t_2 = 265.4°C$

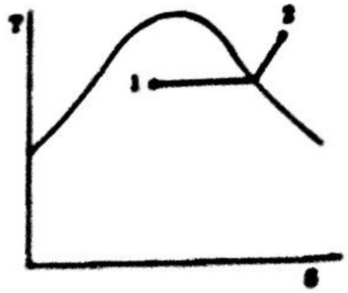
(b) $\Delta u = u_2 - u_1 = 2717.6 - 2286.9 = 430.7$ kJ/kg

(c) $W = p(v_2 - v_1) = 1700\,(0.13796 - 0.09596) = 71.4$ kJ/kg

(d) $\Delta s = s_2 - s_1 = 6.7088 - 5.6761 = 1.0327$ kJ/(kg) (K)

(e) $Q = h_2 - h_1 = 2952.1 - 2450.0 = 502.1$ kJ/kg





(2-2) Steam with an enthalpy of 2843.5 kJ/kg undergoes a constant pressure process at 0.9 MPa until the enthalpy becomes 2056.1 kJ/kg. What are (a) the initial temperature or quality, (b) Δu, (c) W, (d) Δs, and (e) Q.

**SOLUTION:**

Steam with an enthalpy of 2843.5 kJ/kg at a pressure of 0.90 MPa is a superheated vapor.

At $p_1 = 0.90$ MPa

$$23.3\left[9.9\left[\begin{array}{cc} 2833.6 & 200 \\ 2843.5 & t_1 \\ 2856.9 & 210 \end{array}\right] x_1\right] 10 \left[\begin{array}{c} 0.2303 \\ v_1 \\ 0.2364 \end{array}\right] x_2 \Bigg] 0.0061 \left[\begin{array}{c} 2626.3 \\ u_1 \\ 2644.2 \end{array}\right] x_3 \Bigg] 17.9 \left[\begin{array}{c} 6.7522 \\ s_1 \\ 6.8008 \end{array}\right] x_4 \Bigg] 0.0482$$

$$\frac{9.9}{23.3} = \frac{x_1}{10} = \frac{x_2}{0.0061} = \frac{x_3}{17.9} = \frac{x_4}{0.0486}$$

$x_1 = 4.2;\ t_1 = 200 + 4.2 = 204.2°C$

$x_2 = 0.00259;\ v_1 = 0.2303 + 0.00259 = 0.2329$ m³/kg

$x_3 = 7.6;\ u_1 = 2626.3 + 7.6 = 2633.9$ kJ/kg

$x_4 = 0.0206;\ s_1 = 6.7522 + 0.0206 = 6.7728$ kJ/(kg) (K)

Steam with an enthalpy of 2056.1 kJ/kg at a pressure of 0.90 MPa is a wet mixture.

At $p_2 = 0.90$ MPa

$v_{f2} = 0.0011212$ $h_{f2} = 742.83$ $s_{f2} = 2.09046$ $u_{f2} = 741.83$

$v_{fg2} = 0.2139$ $h_{fg2} = 2031.1$ $s_{fg2} = 4.5280$ $u_{fg2} = 1838.6$

$v_{g2} = 0.215$ $h_{g2} = 2773.91$ $s_{g2} = 6.6$ $s_{g2} = 2580.5$

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}} = \frac{2056.1 - 742.83}{2031.1} = 0.6466$$

$v_2 = v_{f2} + x_2 v_{fg2} = 0.0011212 + (0.6466)(0.2139) = 0.1394\ m^3/kg$

$u_2 = u_{f2} + x_2 u_{fg2} = 741{:}83 + (0.6466)(1838.6) = 1930.7\ kJ/kg$

$s_2 = s_{f2} + x_2 s_{fg2} = 2.0946 + (0.6466)(4.5280) = 5.0224\ kJ/(kg)(K)$

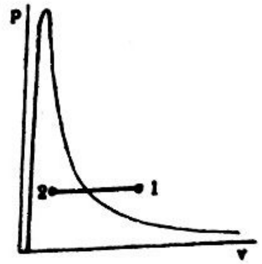
(a) $t_1 = 204.2°C$

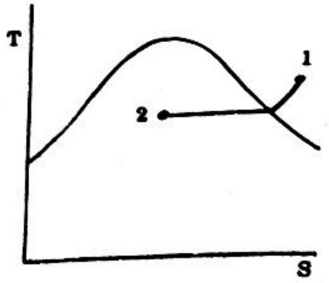
(b) $\Delta u = u_2 - u_1 = 1930.7 - 2633.9 = -703.2\ kJ/kg$

(c) $W = p(v_2 - v_1) = 900\,(0.1394 - 0.2329) = -84.15\ kJ/kg$

(d) $\Delta s = s_2 - s_1 = 5.0224 - 6.7728 = -1.7505\ kJ/(kg)(K)$

(e) $Q = h_2 - h_1 = 2056.1 - 2843.5 = -787.4\ kJ/kg$





## Constant Volume Process

(a) The process on the pv and Ts planes.

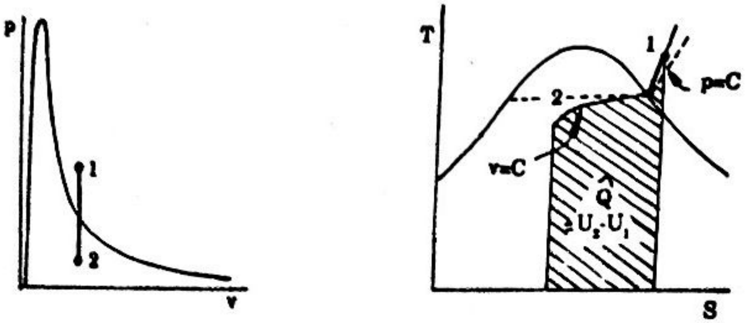


Fig. 2-3. Reversible Constant Volume (Isometric) Process.

Point 1 is in the superheat region and point 2 in the wet region so that,

$$v_1 = v_2 = v_{f2} + x_2 v_{fg2}$$

(b) Work of the nonflow process, Wn.

$$W_n = 0$$

(c) Transferred heat, Q.

$$Q = u_2 - u_1 + W_n \text{ [unit mass]}$$

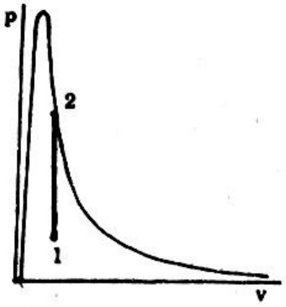
$$Q = u_2 - u_1$$

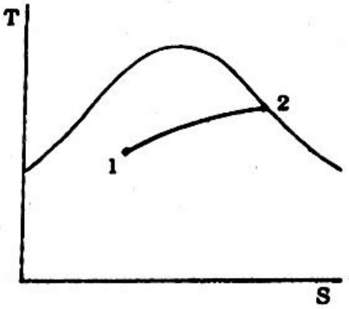
and $u_2 = u_{f2} + x_2 u_{fg2}$

$$\text{or } Q = (h_2 - p_2 v_2) - (h_1 - p_1 v_1)$$

(2-3) One kg of steam at 260°C and with an enthalpy of 1861 kJ/kg is confined in a rigid container. Heat is applied until the steam becomes saturated. Determine (a) Q, (b) Δh, (c) Δs, and (d) the final temperature.

**SOLUTION:**





The enthalpy of saturated vapor at 260°C is 2796.9 kJ/kg, point 1 is a wet-mixture.

At $t_1 = 260°C$

$v_f = 0.0012755$ $\quad u_f = 1128.39$ $\quad h_f = 1134.37$ $\quad s_f = 2.8838$

$v_{fg1} = 0.0409345$ $\quad u_{fg1} = 1470.6$ $\quad h_{fg1} = 1662.5$ $\quad s_{fg1} = 3.1181$

$v_{g1} = 0.04221$ $\quad u_{g1} = 2599.0$ $\quad h_{g1} = 2796.9$ $\quad s_{g1} = 6.0019$

$$x_1 = \frac{h_1 - h_f}{h_{fg1}} = \frac{1861 - 1134.37}{1662.5} = 0.4371$$

$v_1 = v_f + x_1 v_{fg1} = 0.0012755 + (0.4371)(0.0409345) = 0.019168$ m³/kg

$u_1 = u_f + x_1 u_{fg1} = 1128.39 + (0.4371)(1470.6) = 1771.2$ kJ/kg

$s_1 = s_f + x_1 s_{fg1} = 2.8838 + (0.4371)(3.1181) = 4.2467$ kJ/(kg) (K)

$v_2 = v_{g2} = v_1 = 0.019168$ m³/kg

$$\begin{array}{ccccccccccccccc} & & 0.19294 & 307 & & & 2551.8 & & & 2734.3 & & & 5.6479 & & \\ 0.000\,319 & 0.000126 & 0.019168 & t_2 & x_1 & 1.0 & u_2 & x_2 & 1.7 & h_2 & x_3 & 2.3 & s_2 & x_4 & 0.0082 \\ & & 0.018975 & 308 & & & 2550.1 & & & 2732.0 & & & 5.6397 & & \end{array}$$

$$\frac{0.000126}{0.000319} = \frac{x_1}{1} = \frac{x_2}{1.7} = \frac{x_3}{2.3} = \frac{x_4}{0.0082}$$

$x_1 = 0.4;\ t_2 = 307 + 0.4 = 307.4°C$

$x_2 = 0.7;\ u_2 = 2551.8 - 0.7 = 2551.1$ kJ/kg

$x_3 = 0.9;\ h_2 = 2734.3 - 0.9 = 2733.4$ kJ/kg

$x_4 = 0.0032;\ s_2 = 5.6479 - 0.0032 = 5.6447$ kJ/(kg) (K)

(a) $Q = u_2 - u_1 = 2551.1 - 1771.2 = 779.9$ kJ/kg
(b) $\Delta h = h_2 - h_1 = 2733.4 - 1861 = 872.4$ kJ/kg
(c) $\Delta s = s_2 - s_1 = 5.6447 - 4.2467 = 1.398$ kJ/(kg) (K)
(d) $t_2 = 307.4°C$

---

**(2-4)** One kg of steam at 121°C and 10% moisture undergoes a constant volume process until the pressure becomes 0.28 MPa. Determine (a) the final temperature and (b) Q. (c) At what pressure was the final condition saturated?

**SOLUTION:**

$y_1 = 10\%$ $\qquad x_1 = 90\%$

At $t_1 = 121°C$

$v_f = 0.0010612$ $\qquad u_f = 507.74$
$v_{fg1} = 0.8648388$ $\qquad u_{fg1} = 2022.6$
$v_{g1} = 0.8659$ $\qquad u_{g1} = 2530.3$

$v_1 = v_f + x_1 v_{fg1} = 0.0010612 + (0.90)(0.8648388) = 0.7794$ m³/kg

$u_1 = u_f + x_1 u_{fg1} = 507.74 + (0.90)(2022.6) = 2328.1$ kJ/kg

Steam at a pressure of 0.28 MPa and with a specific volume of 0.7794 m³/kg is a superheated vapor.

$p_2 = 0.28$ MPa $\qquad v_2 = v_1 = 0.7794$ m³/kg

$$\begin{array}{cccccccccc} & & 0.7683 & 200 & & & 2651.4 & & \\ 0.0172 & 0.0111 & & & x_1 & 10 & & x_2 & 15.6 \\ & & 0.7794 & t_2 & & & u_2 & & \\ & & 0.7855 & 210 & & & 2667.0 & & \end{array}$$

$$\frac{0.0111}{0.0172} = \frac{x_1}{10} = \frac{x_2}{15.6}$$

$x_1 = 6.5;\ t_2 = 200 + 6.5 = 206.5°C$

$x_2 = 10.1;\ u_2 = 2651.4 + 10.1 = 2661.5$ kJ/kg

(a) $t_2 = 206.5°C$

(b) $Q = u_2 - u_1 = 2661.5 - 2328.1 = 333.4$ kJ/kg

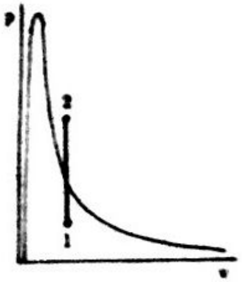
(c) $v_g = 0.7794$ m³/kg

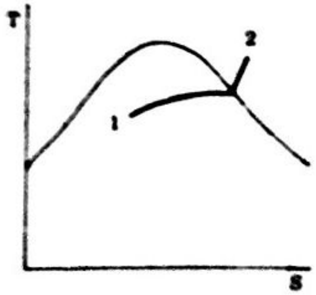
$$0.005\left[x\left[\begin{array}{ll}0.225 & 0.7933\\ p & 0.7794\\ 0.230 & 0.7771\end{array}\right] 0.0139\right] 0.0139$$

$$\frac{x}{0.005} = \frac{0.0139}{0.0162}$$

$$x = 0.00429$$

$$p = 0.225 + 0.00429 = 0.229 \text{ MPa}$$





(2-5) A 0.50-m³ tank contains saturated steam at 300 kPa. Heat is transferred until the pressure reaches 100 kPa. Determine (a) the heat transferred, (b) the final temperature, and (c) the final steam quality

**SOLUTION:**

$p_1 = 0.30$ MPa, saturated vapor

$v_1 = v_g$ at 0.30 MPa $= 0.6058$ m³/kg

$u_1 = u_g$ at 0.30 MPa $= 2543.6$ kJ/kg

$$m = \frac{V_1}{v_1} = \frac{0.50}{0.6058} = 0.8254 \text{ kg}$$

$p_2 = 0.10$ MPa, $v_2 = v_1 = 0.6058$ m³/kg, this is a wet mixture.

$v_{f2} = 0.0010432$ m³/kg $\qquad u_{f2} = 417.36$ kJ/kg

$v_{fg2} = 1.693$ m³/kg $\qquad u_{fg2} = 2088.7$ kJ/kg

$v_{g2} = 1.694$ m³/kg $\qquad u_{g2} = 2506.1$

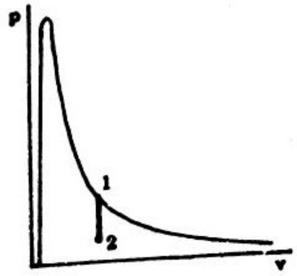
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{0.6058 - 0.0010432}{1.693} = 0.3572$$

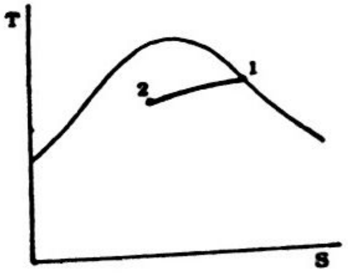
$$u_2 = u_{f2} + x_2 u_{fg2} = 417.36 + (0.3572)(2088.7) = 1163.4 \text{ kJ/kg}$$

(a) $Q = m(u_2 - u_1) = (0.8254)(1163.5 - 2543.6) = -1139.2$ kJ

(b) $t_2 = 99.63$°C

(c) $x_2 = 35.72\%$





(2.6) To what temperature must a rigid tank containing dry saturated steam at 127°C be cooled to yield a mixture of 25% quality?

**SOLUTION:**

$v_1 = v_g$ at 127°C $= 0.7277$ m³/kg

$v_2 = v_1 = 0.7277$ m³/kg $\qquad x_2 = 25\%$

$$v_2 = v_{f2} + x_2 v_{fg2}$$

$$v_2 = v_{f2} + x_2 (v_{g2} - v_{f2})$$

$$v_2 = v_{f2} + x_2 v_{g2} - x_2 v_{f2}$$

$$v_2 = (1 - x_2)v_{f2} + x_2 v_{g2}$$

$$v_2 = y_2 v_{f2} + x_2 v_{g2}$$

$$0.7277 = 0.75\ v_{f2} + 0.25\ v_{g2}$$

Neglecting $v_{f2}$ in comparison with $v_{g2}$;

$$0.25 v_{g2} = 0.7277$$

$$v_{g2} = 2.9108\ \text{m}^3/\text{kg}$$

$$x\left[\,x\left[\begin{matrix}84\\t_2\\85\end{matrix}\right.\right. \qquad \begin{matrix}2.9340\\2.9108\\2.8280\end{matrix}\ \boxed{0.0232}\ \ 0.106$$

$$\frac{x}{1} = \frac{0.0232}{0.106}$$

$x = 0.22$; $t_2 = 84 + 0.22 = 84.22$°C

## Isothermal Process

(a) The process on the pv and Ts planes

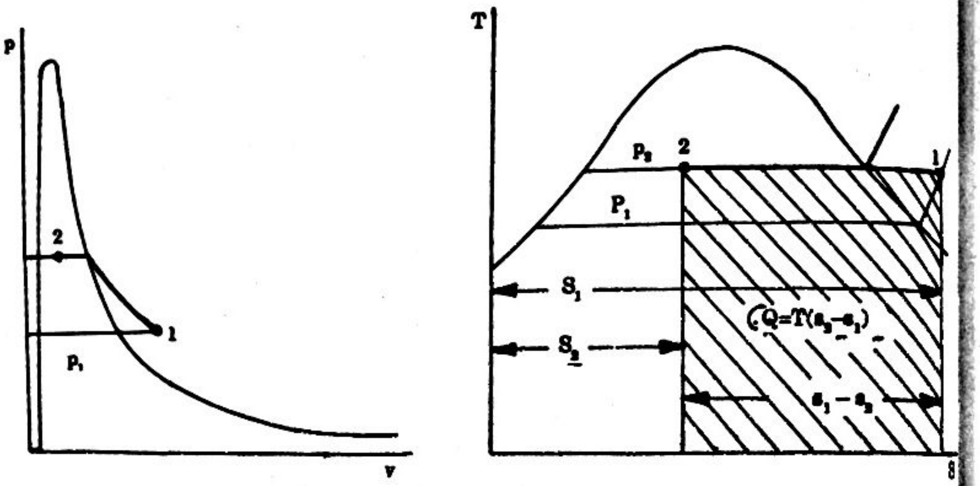


Fig. 2-4. Reversible Isothermal Process.

(b) Work of a nonflow process, $W_n$

From the simple energy equation,

$$W_n = Q - \Delta u \quad [\text{unit mass}]$$

(c) Transferred heat Q from the Ts plane.

$$Q = T\Delta s \quad [\text{unit mass}]$$

$$= T(s_2 - s_1)$$

and

$$s_1 = s_{f1} + x_1 s_{fg1}$$

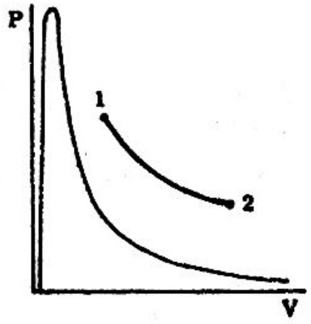
(d) Work of steady flow process, Ws

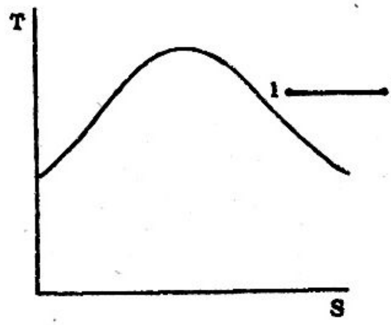
$$Ws = Q - \Delta h - \Delta K$$

(2-7) There are 2.27 kg/min of steam undergoing an isothermal process from 27.5 bar, 316°C to 6.8 bar. Determine (a) ΔS, (b) Q, (c) W for nonflow, and (d) W for steady flow with ΔP = O, ΔK = 42 kJ/min. (1 bar = 100 kPa)

**SOLUTION:**

The initial and final conditions are superheated. The saturation temperature at a pressure of 2.75 MPa is 229°C and at 0.68 MPa is 164°C.





At $p_1 = 2.75$ MPa

$$\begin{array}{cccccccccccc}
 & & 310 & 2774.8 & & & 3025.9 & & & 6.6321 & & \\
10 & 6 & 316 & u_1 & x_1 & 18.6 & h_1 & x_2 & 24.2 & s_1 & x_3 & 0.0412 \\
 & & 320 & 2793.4 & & & 3050.1 & & & 6.6733 & &
\end{array}$$

$$\frac{6}{10} = \frac{x_1}{18.6} = \frac{x_2}{24.2} = \frac{x_3}{0.0412}$$

$x_1 = 11.2$; $u_1 = 2774.8 + 11.2 = 2786$ kJ/kg

$x_2 = 14.5$; $h_1 = 3025.0 + 14.5 = 3040.4$ kJ/kg

$x_3 = 0.0247$; $s_1 = 6.6321 + 0.0247 = 6.6568$ kJ/(kg) (K)

At $p_2 = 0.68$ MPa

$$\begin{array}{cccccccccccc}
 & & 310 & 2815.6 & & & 3080.5 & & & 7.3481 & & \\
10 & 6 & 316 & u_2 & x_1 & 16.0 & h_2 & x_2 & 20.9 & s_2 & x_3 & 0.0355 \\
 & & 320 & 2831.6 & & & 3101.4 & & & 7.3836 & &
\end{array}$$

$$\frac{6}{10} = \frac{x_1}{16} = \frac{x_2}{20.9} = \frac{x_3}{0.0355}$$

$x_1 = 9.6$; $u_2 = 2815.6 + 9.6 = 2825.2$ kJ/kg

$x_2 = 12.5$; $h_2 = 3080.5 + 12.5 = 3093.0$ kJ/kg

$x_3 = 0.0213$; $s_2 = 7.3481 + 0.0213 = 7.3694$ kJ/(kg) (K)

(a) $\Delta S = m(s_2 - s_1) = 2.27\ (7.3694 - 6.6568) = 1.6176$ kJ/ (min) (K)

(b) $Q = T\ (\Delta S) = (316 + 273)\ (1.6176) = 952.8$ kJ/min

(c) $\Delta U = m(u_2 - u_1) = 2.27\ (2825.2 - 2786) = 88.98$ kJ/min

$W_n = Q - \Delta U = 952.8 - 88.98 = 863.8$ kJ/min

(d) $\Delta H = m\ (h_2 - h_1) = 2.27\ (3093 - 3040.4) = 119.4$ kJ/min

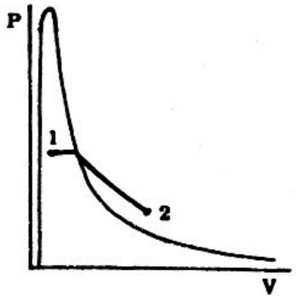
$W_s = Q - \Delta K - \Delta H = 952.8 - 42 - 119.4 = 791.4$ kJ/min

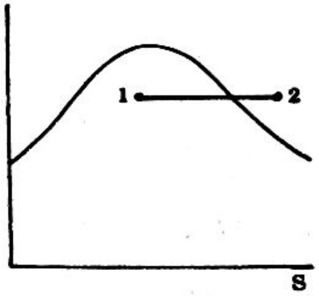
(2-8) Steam at 200°C and with an entropy of 5.6105 kJ/(kg) (K) expands isothermally to 0.515 MPa. For 5 kg (a) what are ΔS, ΔH, and ΔU? Determine Q and W (b) for a nonflow process, (c) for a steady flow process with ΔK = 0.

**SOLUTION:**

Steam with an entropy of 5.6105 kJ/(kg) (K) at 200°C is a wet-mixture.

Steam under a pressure of 0.515 MPa at 200°C is a superheated vapor.





At $t_1 = 200$°C

$v_{f1} = 0.0011565$ $u_{f1} = 850.65$ $h_{f1} = 852.45$ $s_{f1} = 2.3309$

$v_{fg1} = 0.12620$ $u_{fg1} = 1744.7$ $h_{fg1} = 1940.7$ $s_{fg1} = 4.1014$

$v_{g1} = 0.12736$ $u_{g1} = 2595.3$ $h_{g1} = 2793.2$ $s_{g1} = 6.4323$

$$x_1 = \frac{s_1 - s_{f1}}{s_{fg1}} = \frac{5.6105 - 2.3309}{4.1014} = 0.7996$$

$v_1 = v_{f1} + x_1 v_{fg1} = 0.0011563 + (0.7996)(0.12620) = 0.10207$ m³/kg

$u_1 = u_{f1} + x_1 u_{fg1} = 850.65 + (0.7996)(1744.7) = 2245.7$ kJ/kg

$h_1 = h_{f1} + x_1 h_{fg1} = 852.45 + (0.7996)(1940.7) = 2404.2$ kJ/kg

At $t_2 = 200°C$

$$\begin{array}{ccccccccccccccc} & & 0.500 & 0.4249 & & & 2642.9 & & & 2855.4 & & & 7.0592 & & \\ 0.020 & 0.005 & 0.515 & v_2 & x_1 & 0.0168 & u_2 & x_2 & 0.8 & h_2 & x_3 & 1.1 & s_2 & x_4 & 0.0198 \\ & & 0.520 & 0.4081 & & & 2642.1 & & & 2854.3 & & & 7.0394 & & \end{array}$$

$$\frac{0.005}{0.020} = \frac{x_1}{0.0168} = \frac{x_2}{0.8} = \frac{x_3}{1.1} = \frac{x_4}{0.0198}$$

$x_1 = 0.0042$; $v_2 = 0.4081 + 0.0042 = 0.4123$ m³/kg

$x_2 = 0.20$; $u_2 = 2642.1 + 0.20 = 2642.3$ kJ/kg

$x_3 = 0.3$; $h_2 = 2854.3 + 0.3 = 2854.6$ kJ/kg

$x_4 = 0.0045$; $s_2 = 7.0394 + 0.00495 = 7.0444$ kJ/(kg)(K)

(a) $\Delta S = m(s_2 - s_1) = 5(7.0444 - 5.6105) = 7.1695$ kJ/K

$\Delta H = m(h_2 - h_1) = 5(2854.6 - 2404.2) = 2252$ kJ

$\Delta U = m(u_2 - u_1) = 5(2642.3 - 2245.7) = 1983$ kJ

(b) $Q = T(\Delta S) = (473)(7.1695) = 3391.2$ kJ

$W_n = Q - \Delta U = 3391.2 - 1983 = 1408.2$ kJ

(c) $Q = 3391.2$ kJ

$W_s = Q - \Delta H$ $(\Delta K = 0)$

$W_s = 3391.2 - 2252 = 1139.2$ kJ

## Adiabatic Processes of Vapor

(a) The reversible process on the pv and Ts planes.

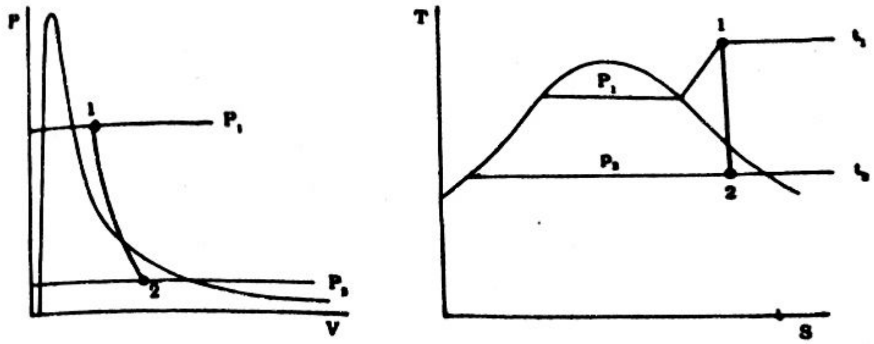


**Fig. 2.5. Reversible Adiabatic Process.**

A reversible adiabatic process is a constant entropy process and also known as "isentropic process"

In an isentropic process, no heat is transferred (Q = 0) and the change in entropy of the substance is also zero ($\Delta S = 0$)

For a nonflow process,

$$Q = \Delta u + W_n = 0 \quad \text{[unit mass]}$$

$$W_n = -\Delta u$$

$$W_n = u_1 - u_2$$

and for steady flow process, with $\Delta P = 0$

$$K_1 + h_1 + Q = K_2 + h_2 + W_s$$

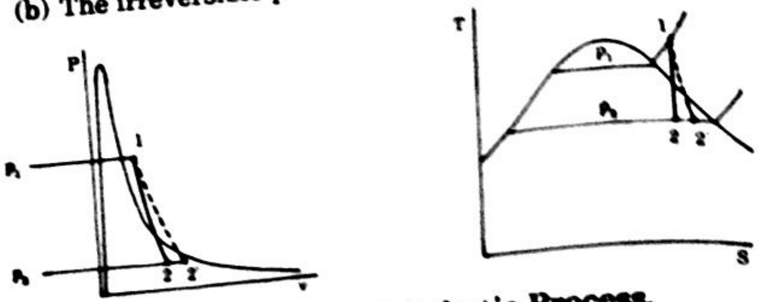
with Q = 0,

$$W_s = h_1 - h_2 - \Delta K$$

with $\Delta K = 0$,

$$W_s = h_1 - h_2$$

(b) The irreversible process on the pv and Ts planes.



F2-6. Irreversible Adiabatic Process.

Process 1-2 is the corresponding reversible adiabatic or isentropic process (ideal expansion)

Process 1-2' is the irreversible adiabatic process (actual expansion).

For a nonflow process with Q = 0,

$$Q = 0 = \Delta u' + W_n \text{ [unit mass]}$$

$$W_n = -\Delta u'$$

where $\Delta u' = u'_1 - u'_2$

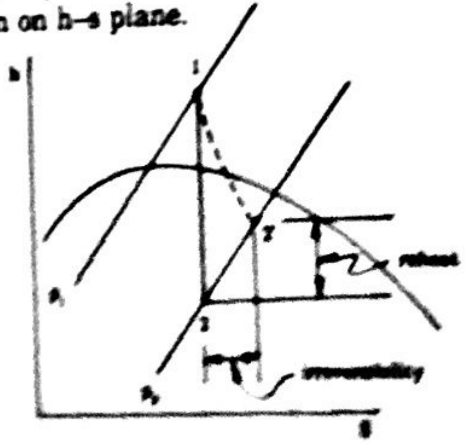
and for steady flow process,

$W_n = h_1 - h_2$, reversible process

$W_n' = h_1 - h_2'$, irreversible process

Efficiency of the process, n

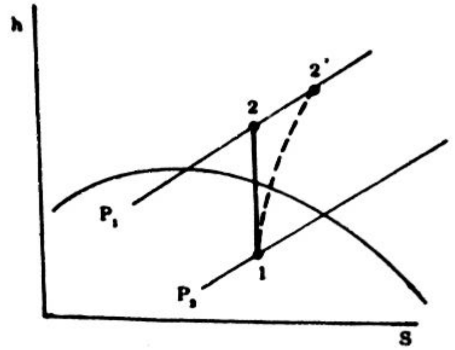
Expansion on h–s plane.



$$n = \frac{\text{actual work}}{\text{ideal work}} = \frac{h_1 - h_2'}{h_1 - h_2}$$

$h_{2'} - h_2$ is known as process internal reheat and $s_{2'} - s_2$ is the irreversibility of the process

Compression on hs plane.

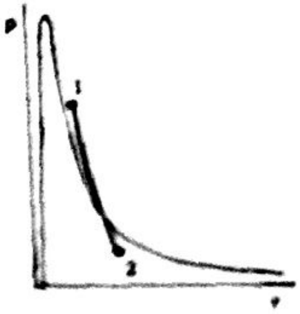


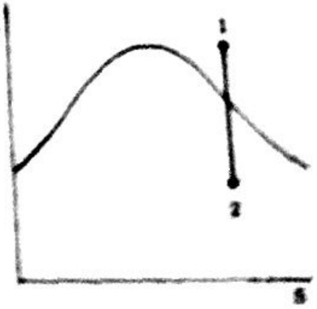
$$\text{Efficiency of compression, } n_1 = \frac{h_1 - h_2}{h_1 - h_{2'}} \times 100\%$$

(2-9) One kg of steam expands isentropically from 2.1 MPa and 374°C to 93°C. Find the final quality and the work for non-flow and steady flow processes.

**SOLUTION:**

The saturation temperature corresponding to a pressure of 2.1 MPa is 214.9°C, therefore, the initial condition is superheated.





At $p_1 = 2.1$ MPa

$$
10\begin{bmatrix} 4\begin{bmatrix}370\\374\end{bmatrix} \\ 380\end{bmatrix}
\begin{bmatrix}\begin{bmatrix}2892.7\\u_1\end{bmatrix}x_1 \\ 2909.8\end{bmatrix}17.1
\begin{bmatrix}\begin{bmatrix}3179.5\\h_1\end{bmatrix}x_2\\3201.7\end{bmatrix}22.2
\begin{bmatrix}\begin{bmatrix}7.0017\\s_1\end{bmatrix}x_3\\7.0360\end{bmatrix}0.0343
$$

$$\frac{4}{10} = \frac{x_1}{17.1} = \frac{x_2}{22.2} = \frac{x_3}{0.0343}$$

$x_1 = 6.8;\ u_1 = 2892.7 + 6.8 = 2899.5$ kJ/kg

$x_2 = 8.9;\ h_1 = 3179.5 + 8.9 = 3188.4$ kJ/kg

$x_3 = 0.0137;\ s_1 = 7.0017 + 0.0137 = 7.0154$ kJ/(kg) (K)

$s_2 = s_1 = 7.0154$ kJ/(kg) (K)

At $t_2 = 93°C$

$h_{f2} = 389.54$ $s_{f2} = 1.2271$ $u_{f2} = 389.46$

$h_{fg2} = 2275.4$ $s_{fg2} = 6.2138$ $u_{fg2} = 2108.7$

$h_{g2} = 2664.9$ $s_{g2} = 7.4409$ $u_{g2} = 2498.2$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{7.0154 - 1.2271}{6.2138} = 0.9315$$

$u_2 = u_{f2} + x_2u_{fg2} = 389.46 + (0.9315)(2108.7) = 2353.7$ kJ/kg

$h_2 = h_{f2} + x_2h_{fg2} = 389.54 + (0.9315)(2275.4) = 2509.1$ kJ/kg

$W_n = u_1 - u_2 = 2899.5 - 2353.7 = 545.8$ kJ/kg

$W_2 = h_1 - h_2 = 3188.4 - 2509.1 = 679.3$ kJ/kg

(2-10) Steam flows isentropically through a nozzle from 1500 kPa, 288°C to 960 kPa. For 454 g/s determine (a) $t_2$, (b) $\Delta V$, (c) W, and (d) $\Delta K$.

**SOLUTION:**

Steam at a temperature of 288°C under a pressure of 1.50 MPa is superheated, since the saturation temperature for 1.50 MPa is 198°C.

At $p_1 = 1.50$ MPa

$$
10\begin{bmatrix} 8\begin{bmatrix}280\\288\end{bmatrix} \\ 290\end{bmatrix}
\begin{bmatrix}\begin{bmatrix}0.16271\\v_1\end{bmatrix}x_1 \\ 0.16621\end{bmatrix}0.0035
\begin{bmatrix}\begin{bmatrix}2748.6\\u_1\end{bmatrix}x_2\\2765.9\end{bmatrix}17.3
\begin{bmatrix}\begin{bmatrix}2992.7\\h_1\end{bmatrix}x_3\\3015.3\end{bmatrix}22.6
\begin{bmatrix}\begin{bmatrix}6.8381\\s_1\end{bmatrix}x_4\\6.8785\end{bmatrix}0.0404
$$

$$\frac{8}{10} = \frac{x_1}{0.0035} = \frac{x_2}{17.3} = \frac{x_3}{22.6} = \frac{x_4}{0.0404}$$

$x_1 = 0.0028;\ v_1 = 0.16271 + 0.0028 = 0.16551$ m³/kg

$x_2 = 13.8;\ u_1 = 2748.6 + 13.8 = 2762.4$ kJ/kg

$x_3 = 18.1;\ h_1 = 2992.7 + 18.1 = 3010.8$ kJ/kg

$x_4 = 0.0323;\ s_1 = 6.8381 + 0.0323 = 6.8704$ kJ/(kg) (K)

The final condition, $p_2 = 0.960$ MPa and $s_2 = 6.8704$ kJ/(kg) (K), is superheated. The entropy of saturated vapor at a pressure of 0.960 MPa is 6.6005 kJ/(kg) (K).

At $p_2 = 0.960$ MPa

$$
0.0441\begin{bmatrix} 0.0115\begin{bmatrix}6.8589 & 230\\6.8704 & t_2\end{bmatrix}x_1 \\ 6.9030 \quad 240\end{bmatrix}10
\begin{bmatrix}\begin{bmatrix}0.2319\\v_2\end{bmatrix}x_2 \\ 0.2374\end{bmatrix}0.0055
\begin{bmatrix}\begin{bmatrix}2676.9\\u_2\end{bmatrix}x_3\\2694.1\end{bmatrix}17.2
\begin{bmatrix}\begin{bmatrix}2899.6\\h_2\end{bmatrix}x_4\\2922.0\end{bmatrix}22.4
$$

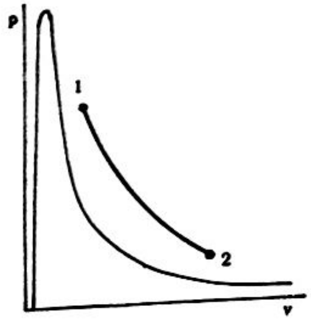
$$\frac{0.0115}{0.0441} = \frac{x_1}{10} = \frac{x_2}{0.0055} = \frac{x_3}{17.2} = \frac{x_4}{22.4}$$

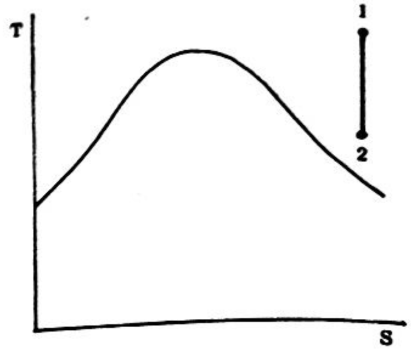
$x_1 = 2.6;\ t_2 = 230 + 2.6 = 232.6°6C$

$x_2 = 0.0014;\ v_2 = 0.2319 + 0.0014 = 0.2333\ m^3/kg$

$x_3 = 4.5;\ u_2 = 2676.9 + 4.5 = 2681.4\ kJ/kg$

$x_4 = 5.8;\ h_2 = 2899.6 + 5.8 = 2905.4\ kJ/kg$





(a) $t_2 = 232.6°C$

(b) $\Delta V = m(v_2 - v_1) = 0.454\ (0.2333 - 0.16551) = 0.0308\ m^3/s$

(c) $W = 0$

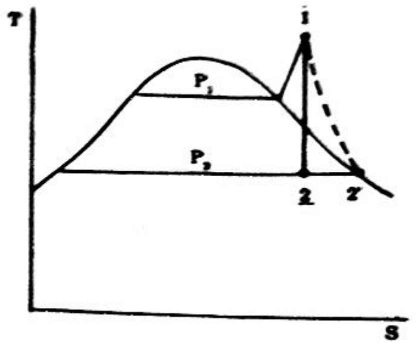
(d) $\Delta H = m(h_2 - h_1) = 0.454\ (2905.4 - 3010.8) = -47.85\ kJ/s$

$$\cancel{Q} = \Delta H + \Delta K + \cancel{W}$$

$$\Delta K = -\Delta H = (-)\ (-47.85) = 47.85\ kJ/s$$

(2-11) A steam turbine receives 0.52 MPa of steam at 300°C. Then expand in an irreversible adiabatic process to a pressure of 0.012 MPa. If the exhaust steam is dry and saturated, calculate (a) the work of an irreversible process and (b) the efficiency of the process.

**SOLUTION:**



**AT $p_1 = 0.52$ MPa and $t_1 = 300°C$**

**$u_1 = 2802.6\ kJ/kg$ $h_1 = 3063.7\ kJ/kg$ $S_1 = 7.4411\ kJ/kg•K$**

**At $p_{2'} = 0.012$ MPa, dry and saturated**

**$h_{2'} = h_g$ at 0.012 MPa = 2591.1 kJ/kg**

**(a) $W_{s'} = h_1 - h_{2'} = 3063.7 - 2591.1 = 472.7\ kJ/kg$**

**(b)** $s_2 = s_{f2} + x_2 s_{fg2}$

$$7.4411 = 0.6923 + x_2\,(7.39)$$

$$x_2 = 0.913$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 206.92 + 0.913\ (2384.1)$$

$$= 2383.6\ kJ/kg$$

$$W_s = h_1 - h_2 = 3063.7 - 2383.6 = 680.1\ kJ/kg$$

$$n = \frac{W_{s'}}{W_s} = \frac{472.7}{680.1} \times 100\% = 69.5\%$$

**Polytropic Process**

(a) The process on the pv plane

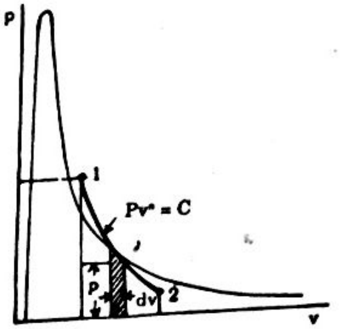


Fig. 2-7. Reversible Polytropic Process.

(b) Work of the nonflow process, $W_n$

Polytropic process is defined by the equation, $pv^n = C$

$$W_n = \int p dv$$

$$W_n = \frac{p_2v_2 - p_1v_1}{1 - n}$$

(c) Transferred heat, Q

$$Q = \Delta u + W_n$$

$$= u_2 - u_1 + \frac{p_2v_2 - p_1v_1}{1 - n}$$

(d) Work of steady flow process, Ws

$$W_s = Q - \Delta h - \Delta K$$

Also, $W_s = -\int v dp = \frac{n(p_2v_2 - p_1v_1)}{1 - n} = (n)(Wn)$

(2-12) Expand 2 kg of steam at 15 bar, 300°C, into the wet region to 100°C in a polytropic process where $pV^{1.21} = C$. Determine (a) $y_2$, (b) $\Delta H$, (c) $\Delta S$, (d) nonflow and steady flow work, and (e) Q.

**SOLUTION:**

For a pressure $p_1 = 1.5$ MPa and temperature $t_1 = 300°C$, the steam is superheated. The saturation temperature for a pressure of 1.5 MPa is 198°C.

$v_1 = 0.16966$ m³/kg $\qquad h_1 = 3037.6$ kJ/kg

$u_1 = 2783.1$ kJ/kg $\qquad s_1 = 6.9179$ kJ/(kg) (K)

At $t_2 = 100°C$, the saturation pressure, $p_2 = 0.10135$ MPa.

$$p_2v_2^n = p_1v_1^n$$

$$v_2 = v_1 \left[\frac{p_1}{p_2}\right]^{\frac{1}{n}} = 0.16966 \left[\frac{1.5}{0.10135}\right]^{\frac{1}{1.21}} = 1.5730 \text{ m}^3/\text{kg}$$

At $t_2 = 100°C$

$v_{f2} = 0.0010435 \qquad u_{f2} = 418.94 \qquad h_{f2} = 419.04 \qquad s_{f2} = 1..3069$

$v_{fg2} = 1.6719 \qquad u_{fg2} = 2087.6 \qquad h_{fg2} = 2257.0 \qquad s_{fg2} = 6.0480$

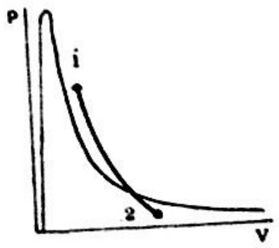
$v_{g2} = 1.6729 \qquad u_{g2} = 2506.5 \qquad h_{g2} = 2676.1 \qquad s_{g2} = 7.3549$

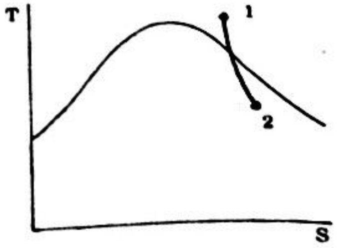
$$x_2 = \frac{v_2 - v_{f2}}{v_{fg2}} = \frac{1.5730 - 0.0010435}{1.6719} = 0.9402$$

$u_2 = u_{f2} + x_2u_{fg2} = 418.94 + (0.9402)(2087.6) = 2381.7$ Kj/kg

$h_2 = h_{f2} + x_2h_{fg2} = 419.04 + (0.9402)(2257.0) = 2541.1$ kJ/kg

$s_2 = s_{f2} + x_2s_{fg2} = 1.3069 + (0.9402)(6.0480) = 6.9932$ kJ/(kg)(K)





(a) $y_2 = 5.98\%$

(b) $\Delta H = m(h_2 - h_1) = 2(2541.1 - 3037.6) = -993$ kJ

(c) $\Delta S = m(s_2 - s_1) = 2(6.9932 - 6.9179) = 0.0753$ kJ/K

(d)
$$W_n = \frac{m(p_2v_2 - p_1v_1)}{1-n} = \frac{2[(0.10135)(1.5730) - (1.5)(0.16966)]}{1 - 1.21}$$

$$= 905.4 \text{ kJ}$$

$$W_s = (n)(W_n) = (1.21)(905.4) = 1095.5 \text{ kJ}$$

(e) $\Delta U = m(u_2 - u_1) = 2(2381.7 - 2783.1) = -802.8$ kJ

$$Q = \Delta U + W_n = -802.8 + 905.4 = 102.6 \text{ kJ}$$

(2-13) Five kg per second of steam at 5.9 MPa and 500°C expand to 0.9 MPa in a polytropic process where $pV^{1.26} = C$. Find (a) $x_2$ or $t_2$, $\Delta H$, $\Delta U$, $\Delta S$, (b) W for nonflow, and (c) W for steady flow if $\Delta K = -40$ kJ/s. (d) Find Q from steady flow and also from nonflow energy equation.

**SOLUTION:**

At a pressure of 5.9 MPa and a temperature of 500°C, the steam is superheated.

$v_1 = 0.05766\ m_3/kg$ $\qquad h_1 = 3423.3$ kJ/kg

$u_1 = 3083.1$ kJ/kg $\qquad s_1 = 6.8892$ kJ/(kg) (K)

$$p_2v_2^n = p_1v_1^n$$

$$v_2 = v_1\left[\frac{p_1}{p_2}\right]^{\frac{1}{n}} = 0.05766\left[\frac{5.9}{0.9}\right]^{\frac{1}{1.26}} = 0.2564 \text{ m}^3\text{/kg}$$

The specific volume of saturated steam at 0.9 MPa is 0.17535 m³/kg. The final condition is a superheated vapor.

At $p_2 = 0.9$ MPa

$$\begin{array}{ccccccccccccccc}
 & & 0.2539 & 2695.9 & & & 2924.4 & & & 6.9364 & & & 240 & & \\
0.0057 & 0.0025 & 0.2564 & u_2 & x_1 & 16.8 & h_2 & x_2 & 21.9 & s_2 & x_3 & 0.0423 & t_2 & x_4 & 10 \\
 & & 0.2596 & 2712.7 & & & 2946.3 & & & 6.9787 & & & 250 & &
\end{array}$$

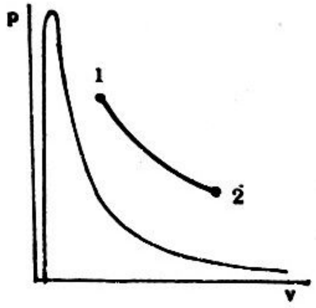
$$\frac{0.0025}{0.0057} = \frac{x_1}{16.8} = \frac{x_2}{21.9} = \frac{x_3}{0.0423} = \frac{x_4}{10}$$

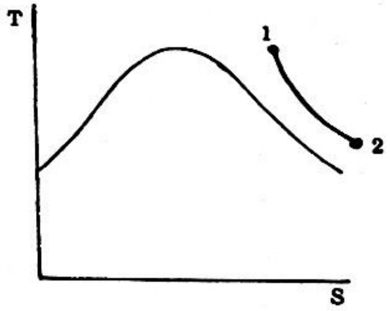
$x_1 = 7.4$; $u_2 = 2695.9 + 7.4 = 2703.3$ kJ/kg

$x_2 = 9.6$; $h_2 = 2924.4 + 9.6 = 2934$ kJ/kg

$x_3 = 0.0186$; $s_2 = 6.9364 + 0.0186 = 6.955$ kJ/kg.K

$x_4 = 4.4$; $t_2 = 240 + 4.4 = 244.4$°C





(a) $t_2 = 244.4$°C

$$\Delta H = m(h_2 - h_1) = 5(2934 - 3423.3) = -2446.5 \text{ kJ/s}$$

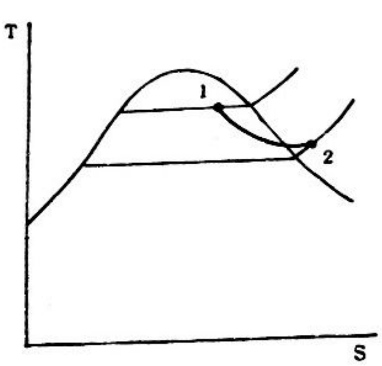
(b) $\Delta s = s_2 - s_1$

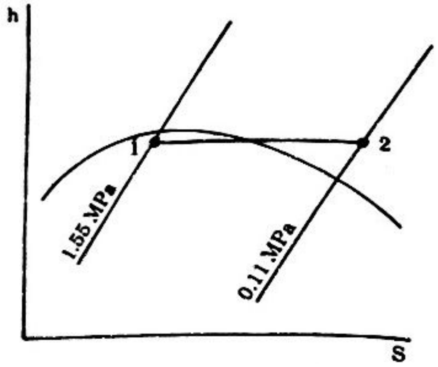
$= 7.143 - 6.8331$

$= 0.31$ kJ/kg•K

(2-15) A throttling calorimeter receives steam from a boiler drum at 0.11 MPa and is superheated by 10 degrees. If the boiler drum pressure is 1.55 MPa, what is the quality of the steam generated by the boiler?

**SOLUTION:**





At $p_2 = 0.11$ MPa, $t_{sat} = 102.31°C$

$t_2 = t_{sat} + °SH$

$= 102.31 + 10$

$= 112.31°C$

$h_2 = 2700$ kJ/kg (by interpolation)

At $p_1 = 1.55$ MPa, $h_{f1} = 851.92$ kJ/kg and $h_{fg1} = 1941.2$ kJ/kg

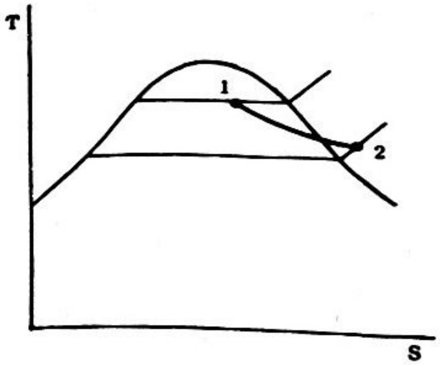
$h_1 = h_{f1} + x_1 h_{fg1}$

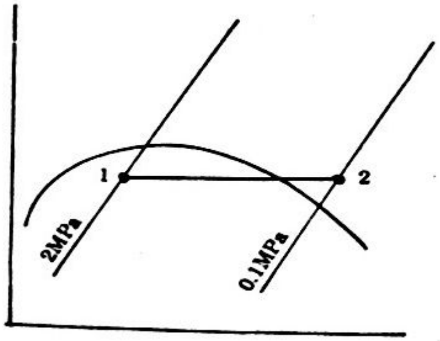
$2700 = 851.92 + x_1\,(1941.2)$

$x_1 = 95.20\%$

(2-16) A steam calorimeter receives steam from a pipe at 0.1 MPa and 20°SH. For a pipe steam pressure of 2 MPa, what is the quality of the steam?

**SOLUTION:**





At $p_2 = 0.1$ MPa, $h_{g2} = 2675.5$ kJ/kg

$h_2 = h_{g2} + c_p\,(°SH)$

$= 2675.5 + (2)\,(20)$

$= 2715.5$

At $p_1 = 2$ MPa, $h_{f1} = 908.79$ kJ/kg and $h_{fg1} = 1890.7$ kJ/kg

$h_1 = h_{f1} + x_1 h_{fg1}$

$2715.5 = 908.79 + x_1\,(1890.7)$

$x_1 = 95.56\%$

## Problems

1. A piston-cylinder containing steam at 700 kPa and 250°C undergoes a constant pressure process until the quality is 70%. Determine per kilogram (a) the work done, (b) the heat transferred, (c) the change of internal energy, and (d) the change of enthalpy. Ans. (a) –101.4 kJ/kg; (b) –810 kJ/kg; (c) –708.5 kJ/kg; (d) –810 kJ/kg

2. Steam at 3.1 MPa and with 74.28°C SH rejects 1973.36 kJ per kg of steam heat at constant pressure, determine (a) the final temperature and (b) the change in specific entropy. Ans. (a) 235.7°C; (b) –3.8572 kJ/kg•K

3. With 3% moisture, 2.5 kg of steam has an enthalpy of 6600 kJ. It is heated at constant pressure to a final condition of 80° superheat. Find (a) the pressure of the process and (b) the quantity of heat received by the steam. Ans. (a) 0.22 MPa; (b) 575 kJ.

4. Five kg of water vapor are contained at 150 kPa and 90 per cent quality in a suitable enclosure. Calculate the heat which must be added in order to just produce a saturated vapor. What will the pressure be at the end of the heating process? Ans. 1045.5 kJ, 168 kPa.

5. A rigid vessel contains 5 kg of wet steam at 0.4 MPa. After the addition of 9585 kJ the steam has a pressure of 2.0 MPa and a temperature of 700°C. Determine the initial internal energy and the specific volume of the steam. Ans. 1553.9 kJ/kg, 0.2258 m³/kg.

6. A 1-kg steam-water mixture at 1.0 MPa is contained in an inflexible tank. Heat is added until the pressure rises to 3.5 MPa and the temperature to 400°C. Determine the heat added. Ans. 1378.7 kJ.

7. Water vapor at 100 kPa and 150°C is contained in a rigid vessel. At what temperature will the vapor start to condense when the container is cooled slowly? How much heat will have to be removed per kilotram in this cooling process? Ans. 95.68°C, –81.4 kJ/kg

8. Water at 2.5 MPa and 200°C is heated at constant temperature up to a quality of 80%. Find (a) the quantity of heat received by the water, (b) the change in internal energy, and (c) the work of a nonflow process. Ans. (a) 2025.7 kJ/kg; (b) 1396.5 kJ/kg; (c) 629.17 kJ/kg.

9. Three kilograms of steam initially at 2.5 MPa and a temperature of 370°C have 2460 kJ of heat removed at constant temperature until the quality is 90%. Determine (a) the pressure when dry saturated steam exists and (b) the work. Ans. (a) 21.03 MPa; (b) –4550.7 kJ/

10. Water vapor at 100 KPa and 150°C is compressed isothermally until half the vapor has condensed. How much work must be performed on the steam in this compression process per kilogram? Ans. –1384.7 kJ.

11. A steam engine isentropically expands 5 kg/sec of steam from 0.68 MPa, 220°C. the exhaust is dry and saturated. Determine (a) the work of a nonflow process (b) the finaltemperature, and (c) the work of a steady flow process. Ans. (a) 641.05 kJ/sec; (b) 131.9°C/ (c) 838.38 kJ.

12. Three kg of steam has an initial enthalpy of 7200 kJ at 0.145 MPa. Isentropic compression gives a final pressure of 0.29 MPa. Find (a) the initial and final temperatures and (b) the work of a steady and nonflow processes. Ans. (a) 110,4°C, 132.4°C; (c) –288.5 kJ, –325.8 kJ.

13. Saturated steam at 0.16 MPa is compressed in an irreversible adiabatic process with an efficiency of 80.24%. For a final pressure of 0.45 MPa, determine (a) the final and initial entropies and (b) the ideal and actual temperatures of compressed steam. Ans. (a) 7.2 kJ/kg•K, 7.3 kJ/kg•K; (b) 220°C, 244°C.

14. Wet steam at 200°C and 20% moisture is compressed in an irreversible adiabatic process to a pressure of 2.30 MPa and a temperature of 225°C. Determine (a) the work of compression, (b) the compression efficiency, (c) the irreversibility of the process, and (d) the compression internal reheat. Ans. (a) –413.49 kJ/kg; (b) 33.3%; (c) 0.71018 kJ/kg•K; (d) 275.73 kJ/kg.

15. Steam at 5 MPa and 400°C expands polytropically to 1.0 MPa according to $pV^{1.3} = C$. Determine the work of nonflow and steady flow, the heat transferred, Δh, and Δs. Ans. 298.9 kJ/kg, 388.6 kJ/kg, –7.6 kJ/kg, –396.2 kJkg, –0.0127 kJ/kg•K.

16. Determine the heat and the work of the polytropic expansion of steam from 2.8 MPa at 340°C to 0.76 MPa at 200°C. Ans. –3000 kJ/kg, –3012.5 kJ/kg

17. Steam at 173.46°C and with a specific entropy of 6 kJ/kg•K undergoes a polytropic process with n = 1.18 until the

pressure becomes 2 MPa. (a) What is the final temperature of the steam? (b) What is the work and the quantity of heat of the process? Ans. (a) 212.42°C; (b) –126.79 kJ/kg, (c) –926.79 kJ/kg.

18. Wet steam at 1 MPa flowing through a pipe is throttled to a pressure of 0.1 MPa. If the throttling temperature is 110°C, what is the quality of the steam in the pipe? Ans. 96%.

19. Steam is throttled to 0.1 MPa with a 20 degrees of superheat. (a) What is the quality of throttled steam if its pressure is 0.75 MPa/ (a) What is the enthalpy of the process? Ans. (a) 97.6%; (b) 2713 kJ/kg.

# 3 Power Cycles

## The Rankine Cycle

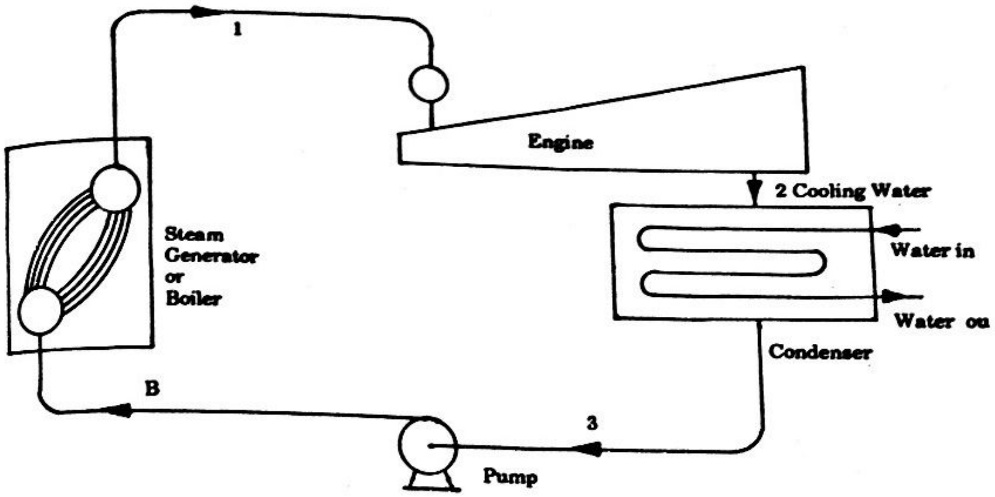

1
Engine
2 Cooling Water
Water in
Water ou
Condenser
Steam
Generator
or
Boiler
B
3
Pump

Fig. 3-1. Diagrammatic Layout for Steam Power Plant

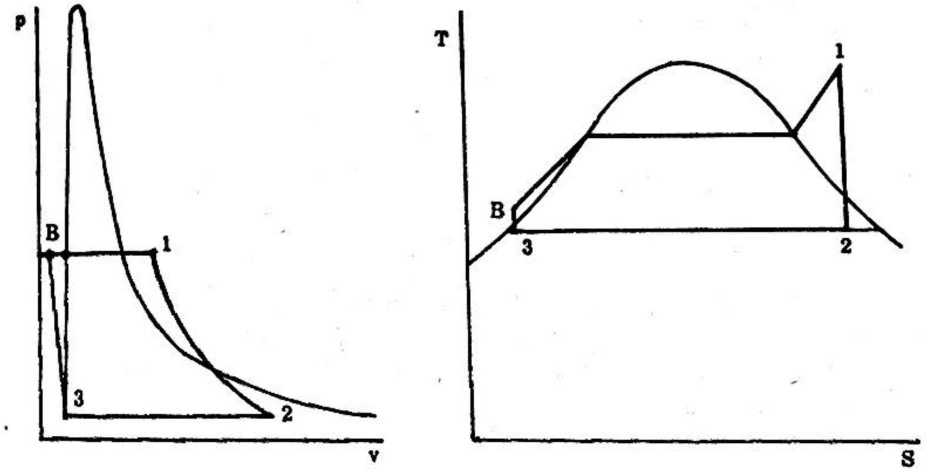

P
B
1
3
2
v
T
1
B
3
2
S

Fig. 3-2. Rankine Cycle

**The ideal Rankine cycle is composed of the following processes:**

1-2: isentropic expansion in the engine, s = C
2-3: constant pressure rejection of heat in the condenser, p = c
3-B: adiabatic pumping, s = C
B-1: constant pressure addition of heat in the steam generator, p = C

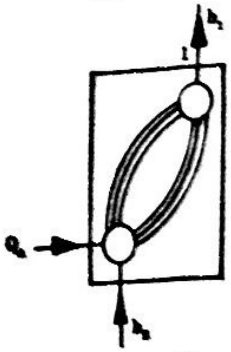
NOTE:

a. In the ideal cycle, the state of steam leaving the steam generator and entering the engine are the same as well as the state of feedwater leaving the pump and entering the steam generator. This means that there is no pressure drop and no heat leakage in the steam line and feedwater line.

b. The quantity of the working substance within the system is constant. This implies that there are no leakages in the system.

Cycle Analysis

A. Heat added, $Q_A$



Steam Generator

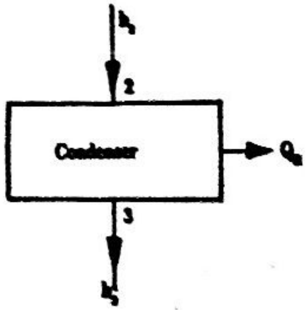
Energy balance:

$$E_{in} = E_{out}$$

$$Q_A + h_B = h_1$$

$$Q_A = h_1 - h_B$$

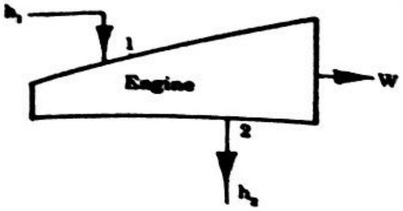
B. Heat Rejected, $Q_R$



Energy balance:

$$E_{in} = E_{out}$$
$$h_2 = h_3 + Q_R$$
$$Q_R = h_2 - h_3$$

C. Engine Work, W



Energy balance:

$$E_{in} = E_{out}$$

$$h_1 = h_2 + W$$

$$W = h_1 - h_2$$

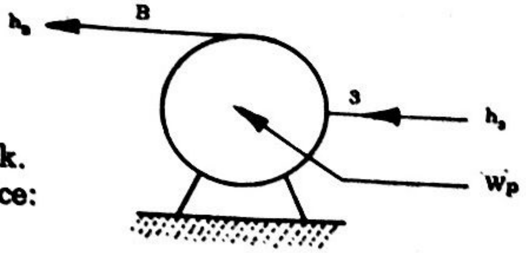
Considering the change in kinetic energy,

$$h_1 + K_1 = h_2 + K_2 + W$$

$$W = h_1 - h_2 + K_1 - K_2$$

$$W = h_1 - h_2 + \frac{\upsilon_1^2 - \upsilon_2^2}{2k}$$

D. Pump Work, $W_p$



1. Exact Pump Work.
   Energy balance:

$$E_{in} = E_{out}$$

$$h_3 + W_p = h_B$$
$$W_p = h_B - h_3$$

2. Approximate Pump Work

The state of feedwater leaving the pump is that of a compressed liquid. Very often, compressed liquid tables are not available, hence, the properties of a compressed liquid are not easily obtainable. Therefore, the exact pump work is difficult to determine.

The following assumptions are made in the determination of the approximate pump work.

(a) Water is "practically" an incompressible liquid.
Therefore, $v_3 = v_B$

(b) The change in internal energy is negligible.
$u_B = u_3$

Energy balance:

$$E_{in} = E_{out}$$

$$u_3 + W_{f3} + W_p = u_B + W_{fB}$$

$$W_p = W_{fB} - W_{f3}$$

$$W_p = p_B v_B - p_3 v_3$$

$$W_p = v_3 (p_B - p_3)$$

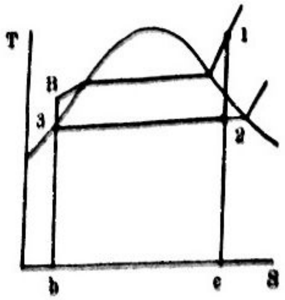
E. Net Cycle Work, $W_{net}$

$$W_{net} = \text{Gross work} - \text{Pump work}$$

$$W_{net} = W - W_p$$

$$W_{net} = h_1 - h_2 - W_p$$

Another method of determining the net cycle work is obtaining it from the Ts diagram.



Rankine Cycle

$$W_{net} = \text{area } 1-2-3-B-1$$
$$= \text{area } (1-c-b-B-1) - \text{area } (2-3-b-c-2)$$

$$= Q_A - Q_R$$
$$= (h_1 - h_B) - (h_2 - h_3)$$
$$= h_2 - h_1 - (h_B - h_3)$$
$$= h_1 - h_2 - W_p$$

F. **Thermal Efficiency, $e_e$**

$$e_e = \frac{W_{net}}{Q_A} = \frac{h_1 - h_2 - W_p}{h_1 - h_3 - W_p}$$

G. **Steam Rate, m**

Steam rate is the mass of steam used to perform a unit work or the mass flow rate of steam consumed to produce a unit of power. For good design, a lower value of steam rate is desired. A lower value of steam rate means that a smaller quantity of steam is needed to develop the desired power output.

Let P = power output, kW
W = work done by a kg of steam, kJ/kg
m = steam rate, kg/kwh

By definition

$$\text{steam rate} = \frac{\text{mass flow rate}}{\text{power output}}$$
$$= \frac{\text{mass flow rate, kg/h}}{\text{power, kw}}$$

$$\text{but Power} = (\text{mass flow rate}, \frac{\text{kg}}{\text{h}})(W, \frac{\text{kJ}}{\text{kg}})(\frac{1\text{ h}}{3600\text{ s}})$$

$$= \frac{(\text{mass flow rate})(W)}{3600}\text{ kw}$$

$$\text{Then, } m = \frac{\text{mass flow rate, }\frac{\text{Kg}}{\text{h}}}{\frac{(\text{mass flow rate})(W)\text{, kw}}{3600}}$$

$$m = \frac{3600}{W}, \frac{kg}{kwh}$$

For the ideal Rankine cycle

$$m_c = \frac{3600}{W_{net}}, \frac{kg}{kwh}$$

### The Ideal Rankine Engine

The ideal Rankine engine is either a steam turbine or a steam engine. The difference between an ideal Rankine engine and an ideal Rankine cycle is that an ideal engine does not include pump work since it is concerned only with all the processes occuring inside the engine. On the other hand, the ideal cycle must include pump work because the pump is needed for the completion of the cycle. AND FOR THE ENGINE TO BE IDEAL, THE EXPANSION PROCESS SHOULD BE ISENTROPIC.

### Engine Analysis

**A. Work, W**

$$W = h_1 - h_2$$

**B. Thermal Efficiency, $e_e$**

The definition of thermal efficiency does not directly applies to an engine because no heat is added to it. Instead it is charged with the enthalpy of steam entering the engine and credited with the enthalpy of saturated liquid at the condensing temperature.

When applied to an engine the thermal efficiency, $e = W/Q_A$ becomes $e = W/E_c$ where $E_c$ is energy chargeable against the engine.

$E_c$ = enthalpy of steam entering the engine – enthalpy of saturated liquid at the condensing temperature

$$e_e = \frac{W}{E_c} = \frac{h_1 - h_2}{h_1 - h_{f3}}$$

**C. Steam Rate, $m_e$**

$$m_e = \frac{3600}{W} = \frac{\text{throttle steam flow rate}}{\text{engine power output}}$$

**D. Heat Rate, HR**

Heat rate is the energy chargeable per unit of work or the rate of energy chargeable per unit of power.

$$HR = \frac{\text{Energy Rate}}{\text{Power}}$$

$$= \frac{(\text{mass flow rate}, \frac{kg}{h})(E_c, \frac{kJ}{kg})}{(\text{mass flow rate}, \frac{kg}{h})(W, \frac{kJ}{kg})(1 \frac{h}{3600\ s})}$$

$$= (\frac{3600}{W})(Ec)$$

$$= (m_e)(E_c)$$

$$= m_e (h_1 - h_{f3})$$

**E. Relation between $e_e$ and HR.**

$$e_e = \frac{W}{E_c}$$

$$E_c = \frac{(HR)(W)}{3600}$$

Then,

$$e_e = \frac{W}{\frac{(HR)(W)}{3600}}$$

$$e_e = \frac{3600}{HR}$$

As can be seen from the resulting equation, the thermal efficiency is inversely proportional to the heat rate. This means that the lower the heat rate the higher the thermal efficiency and the higher the heat rate the lower the thermal efficiency.

## The Actual Rankine Cycle

Any presence of the following conditions will transform the ideal cycle into an actual one.

a. Pressure drop in the steam generator.
b. Pressure drop in the steam line (1 – 1').
c. Pressure drop in the condenser.
d. Pressure drop in the feedwater line (B – B').
e. Heat losses in the steam line.
f. Heat losses in the turbine/engine.
g. Irreversible adiabatic expansion in the turbine.
e. Inefficient pump.
i. Subcooled condensate.

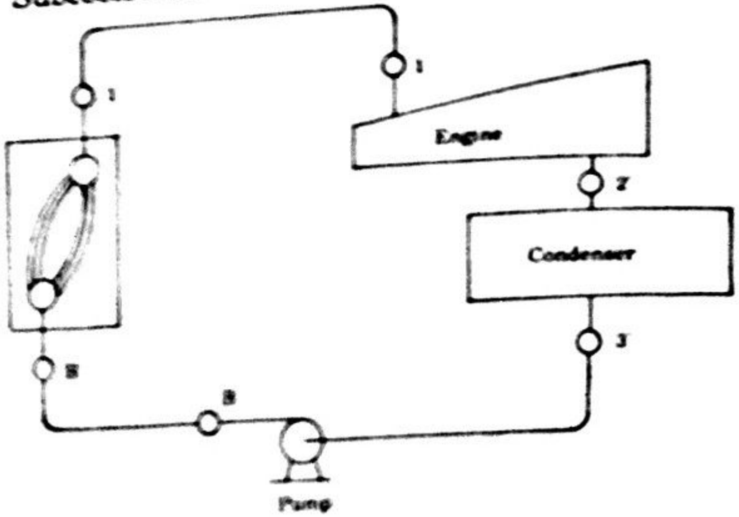


Fig. 3-3. Actual Rankine System

### Cycle Analysis

A. Heat Added, $Q_A$

$$Q_A = h_1 - h_B$$

B. **Heat Rejected, $Q_{R'}$**

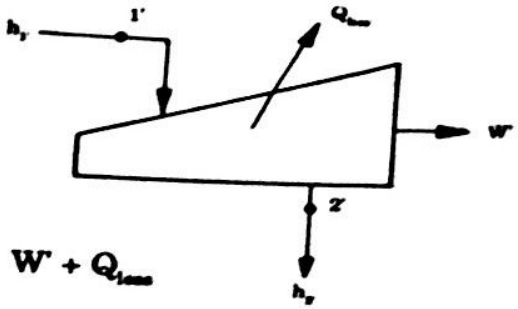
$$Q_{R'} = h_{2'} - h_{3'}$$

C. **Engine Work, W'**

1. **Irreversible adiabatic expansion from 1' – 2'**

$$W' = h_{1'} - h_{2'}$$

2. **Polytropic expansion from 1' – 2'**



$$E_{in} = E_{out}$$

$$h_{1'} = h_{2'} + W' + Q_{loss}$$

$$W' = h_{1'} - h_{2'} - Q_{loss}$$

D. **Pump Work, $W_{p'}$**

$$W_{p'} = \frac{\text{ideal pump work}}{n_m}$$

where: $n_m$ = pump mechanical efficiency

E. **Actual Cycle Thermal Efficiency, $e_{c'}$**

$$e_{c'} = \frac{W' - W_{p'}}{Q_{A'}}$$

## The Actual Rankine Engine

If during the expansion process the steam undergoes a process other than isentropic process, the engine is said to be an actual one.

## Engine Analysis

A. Work, W'

$$W' = h_{1'} - h_{2'}$$

B. Energy Chargeable Against the Engine, $E_{e'}$

$$E_{e'} = h_{1'} - h_{f2'}$$

C. Thermal Efficiency, $e_{e'}$

$$e_{e'} = \frac{W'}{E_{e'}} = \frac{h_{1'} - h_{2'}}{h_{1'}\ h_{f2'}}$$

## Definition of Terms

*Turbogenerator* is a generator driven by a turbine.

*Ideal work, W,* is the work done by the steam during a reversible adiabatic expansion process in the turbine.

*Indicated or actual fluid work, $W_I$* or W', is the work done by the steam during an irreversible adiabatic expansion or polytropic expansion process in the turbine.

*Brake work $W_B$,* is the useful work, i.e., the available work at the engine shaft.

*Combined work, $W_K$,* is the electrical energy available at the generator outlet.

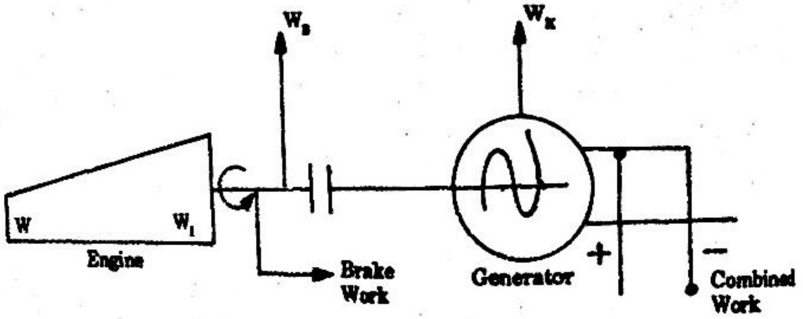


Fig. 3-4. Turbine-Generator Set

## Thermal Efficiencies

a. Ideal Thermal Efficiency, e

$$e = \frac{W}{E_e}$$

b. Indicated Thermal Efficiency, $e_i$

$$e_i = \frac{W_I}{E_e}$$

c. Brake Thermal Efficiency, $e_b$

$$e_b = \frac{W_B}{E_e}$$

d. Combined Thermal Efficiency, $e_k$

$$e_k = \frac{W_k}{E_e}$$

Note: $e > e_i > e_b > e_k$

## Steam Rates

a. Ideal Steam Rate, m

$$m = \frac{3600}{W}$$

b. Indicated Steam Rate, $m_i$

$$m_i = \frac{3600}{W_I}$$

c. Brake Steam Rate, $m_b$

$$m_b = \frac{3600}{W_B}$$

d. Combined Steam Rate, $m_k$

$$m_k = \frac{3600}{W_K}$$

**Engine Efficiencies**

a. **Indicated Engine Efficiency, $n_i$**

$$n_i = \frac{W_I}{W}$$

b. **Brake Engine Efficiency, $n_b$**

$$n_k = \frac{W_B}{W}$$

c. **Combined Engine Efficiency, $n_k$**

$$n_k = \frac{W_K}{W}$$

d. **Turbine/Engine Mechanical Efficiency, $n_{me}$**

$$n_{me} = \frac{W_B}{W_I}$$

e. **Generator Efficiency, $n_g$**

$$n_g = \frac{W_K}{W_B}$$

Heat Rates

a. Ideal Heat Rate, HR

$$HR = (m)(E_c)$$

b. Indicated Heat Rate, $HR_i$

$$HR_i = (m_i)(E_c)$$

c. Brake Heat Rate, $HR_b$

$$HR_b = (m_b)(E_c)$$

d. Combined Heat Rate, $HR_k$

$$HR_k = (m_k)(Ec)$$

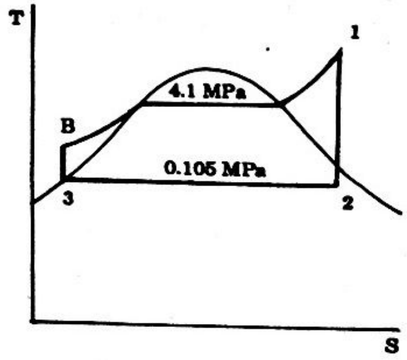
**(3-1)** Steam is generated at 4.10 MPa and 440°C and condensation occurs at 0.105 MPa.

(a) For a Rankine engine operating between these limits, compute the thermal efficiency and the heat rate.

(b) Considering that a Rankine cycle occurs between the same limits, determine $Q_A$, $Q_R$, $W_{net}$, and $e_c$.

(c) What mass flow rate is required for a net output of 30,000 kW?

**SOLUTION:**



$p_1$ = 4.10 MPa  $h_1$ = 3305.7 kJ/kg

$t_1$ = 440°C  $s_1$ = 6.8911 kJ/(kg) (K)

At $p_2$ = 0.105 MPa

$h_{f2}$ = 423.24 kJ/kg  $s_{f2}$ = 1.3181 kJ/(kg) (K)

$h_{fg2}$ = 2254.4 kJ/kg  $s_{fg2}$ = 6.0249 kJ/(kg) (K)

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.8911 - 1.3181}{6.0249} = 0.925$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 423.24 + (0.925)(2254.4) = 2508.6 \text{ kJ/kg}$$

$$h_3 = h_f \text{ at } 0.105 \text{ MPa} = 423.24 \text{ kJ/kg}$$

$$V_{f3} = v_f \text{ at } 0.105 \text{ MPa} = 0.0010443 \text{ m}^3\text{/kg}$$

$W_p = v_{f3}(p_B - p_3) = (0.0010443)(4100 - 105) = 4.17$ kJ/kg

$h_B = h_3 + W_p = 423.24 + 4.17 = 427.4$ kJ/kg

(a) Rankine engine

$W = h_1 - h_2 = 3305.7 - 2508.6 = 797.1$ kJ/kg

$E_c = h_1 - h_{f2} = 3305.7 - 423.24 = 2882.5$ kJ/kg

$$e = \frac{W}{E_c} = \frac{797.1}{2882.5} = 0.2765 \text{ or } 27.65\%$$

$$m = \frac{3600}{W} = \frac{3600}{797.1} = 4.516 \text{ kg/kwh}$$

Heat rate $= (m)(h_1 - h_{f2}) = (4.516)(2882.5) = 13{,}017$ kJ/kwh

(b) Rankine cycle

$Q_A = h_1 - h_B = 3305.7 - 427.4 = 2878.3$ kJ/kg

$Q_R = h_2 - h_3 = 2508.6 - 423.24 = 2085.4$ kJ/kg

$W_{net} = Q_A - Q_R = 2878.3 - 2085.4 = 792.9$ kJ/kg

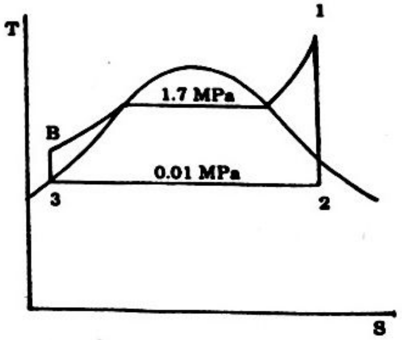
or $W_{net} = W - W_p = 797.1 - 4.17 = 792.9$ kJ/kg

$$e = \frac{W_{net}}{Q_A} = \frac{792.9}{2878.3} = 0.2755 \text{ or } 27.55\%$$

(c) Steam flow rate $= \dfrac{30{,}000 \text{ kW}}{W_{net} \text{ kJ/kg}} = \dfrac{30{,}000 \text{ kJ/s}}{792.9 \text{ kJ/kg}} = 37.84$ kg/s

(3-2) A turbo generator has a combined steam rate of 5.35 kg/kwh at its rated output of 20,000 kW. The steam is at 1.7 MPa, 300 C, and the exhaust is at 0.01 MPa. Calculate (a) the combined heat rate, (b) the combined thermal efficiency, and (c) the combined engine efficiency.

**SOLUTION:**



$p_1 = 1.7$ MPa $\quad h_1 = 3032.1$ kJ/kg $\quad m_k = 5.35$ kg/kwh

$t_1 = 300°\text{C} \quad s_1 = 6.8528$ kJ/(kg) (K)

At $p_2 = 0.01$ MPa

$h_{f2} = 191.83$ kJ/hg $\quad s_{f2} = 0.6493$ kJ/(kg) (K)

$h_{fg2} = 2392.8$ kJ/kg $\quad s_{fg2} = 7.5009$ kJ/(kg) (K)

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.8528 - 0.6493}{7.5009} = 0.827$$

$h_2 = h_{f2} + x_2 h_{fg2} = 191.83 + (0.827)(2392.8) = 2170.7$ kJ/kg

(a) Combined heat rate $= (m_k)(h_1 - h_{f2})$

$= (5.35)(3032.1 - 191.83) = 15{,}195$ kJ/kwh

(b) $W_k = \dfrac{3600}{m_k} = \dfrac{3600}{5.35} = 672.9$ kJ/kg

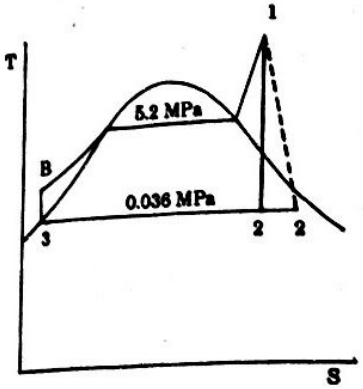
$$e_k = \frac{W_k}{h_1 - h_{f2}} = \frac{672.9}{3032.1 - 191.83} = 0.2369 \text{ or } 23.69\%$$

(c) $W = h_1 - h_2 = 3032.1 - 2170.7 = 861.4$ kJ/kg

$$n_k = \frac{W_k}{W} = \frac{672.9}{861.4} = 0.7812 \text{ or } 78.12\%$$

(3-3) Steam at 5.2 MPa, 400°C expands in a Rankine turbine to 0.036 MPa. For 136 kg/s of steam, determine the work, the thermal efficiency, and the steam rate (a) for the cycle, (b) for the turbine. (c) For an actual turbine with the same specifications, the brake steam rate is 4.80 kg/kwh and the driven electric generator has an efficiency of 93%. Find $e_b$, $n_b$, $W_k$, and quality or temperature of actual exhaust steam.

**SOLUTION:**



$p_1 = 5.2$ MPa $\qquad h_1 = 3192.0$ kJ/kg

$t_1 = 400°C \qquad s_1 = 6.6236$ kJ/(kg) (K)

At $p_2 = 0.036$ MPa

$h_{f2} = 307.05$ kJ/kg $\qquad s_{f2} = 0.9956$ kJ/(kg) (K)

$h_{fg2} = 2325.6$ kJ/kg $\qquad s_{fg2} = 6.7104$ kJ/(kg) (K)

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.6236 - 0.9956}{6.7104} = 0.8387$$

$h_2 = h_{f2} + x_2 h_{fg2} = 307.05 + (0.8387)(2325.5) = 2257.4$ kJ/kg

$h_3 = h_f$ at 0.036 MPa = 307.05 kJ/kg

$v_{f3} = v_f$ at 0.036 MPa = 0.0010249 m³/kg

$W_p = v_{f3}(p_B - p_3) = (0.0010249)(5200 - 36) = 5.29$ kJ/kg

$h_B = h_3 + W_p = 307.05 + 5.29 = 312.3$ kJ/kg

(a) Rankine cycle

$W_{net} = h_1 - h_2 - W_p = 3192.0 - 2257.4 - 5.29 = 929.3$ kJ/kg

$= (929.3)(136) = 126{,}385$ kJ/s or 126,385 kW

$Q_A = h_1 - h_B = 3192.0 - 312.3 = 2879.7$ kJ/kg

$$e = \frac{W_{net}}{Q_A} = \frac{929.3}{2879.7} = 0.3227 \text{ or } 32.27\%$$

$$m = \frac{3600}{W_{net}} = \frac{3600}{929.3} = 3.874 \text{ kg/kwh}$$

(b) Rankine engine

$W = h_1 - h_2 = 3192 - 2257.4 = 934.6$ kJ/kg

$= (934.6)(136) = 127{,}106$ kJ/s or 127,106 kW

$$e = \frac{W}{h_1 - h_{f2}} = \frac{934.6}{3192.0 - 307.05} = 0.324 \text{ or } 32.4\%$$

$$m = \frac{3600}{W} = \frac{3600}{934.6} = 385 \text{ kg/kwh}$$

(c) Actual engine

$$W_B = \frac{3600}{m_b} = \frac{3600}{4.80} = 750 \text{ kJ/kg}$$

$$e_b = \frac{W_B}{h_1 - h_{f2}} = \frac{750}{3192.0 - 307.05} = 0.26 \text{ or } 26\%$$

$n_b = \frac{W_B}{W} = \frac{750}{934.6} = 0.8025$ or 80.25%

$W_k = (W_B)$ (generator efficiency) $= (750)(0.93)$

$= 697.5$ kJ/kg

$= (697.5)(136) = 94{,}860$ kJ/s or 94,860 kW

$h_{2'} = h_1 - W_B = 3192.0 - 750 = 2442$ kJ/kg

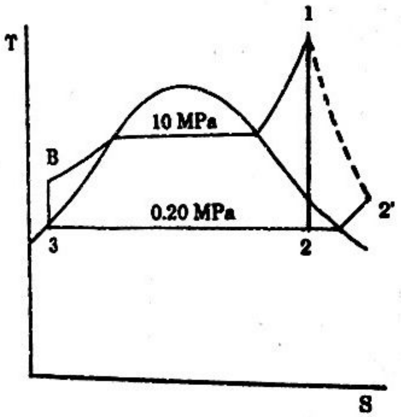
$h_{2'} = h_{f2'} + x_{2'}h_{fg2'}$

$2442 = 307.05 + x_{2'}(2325.5)$

$x_{2'} = 0.9181$ or 91.81%

(3-4) A turbine receives steam at 10 MPa, 600°C and exhaust it at 0.2 MPa. (a) For the ideal Rankine engine, determine the work, steam rate, thermal efficiency, and mean effective pressure. (b) For the actual engine, the brake engine efficiency is 84%; the driven generator efficiency is 93% and the rated output of the generator is 30 MW. Estimate the enthalpy and quality (or temperature) of the exhaust. Compute the combined work, combined heat rate, and the total throttle flow for the rated power.

**SOLUTION:**



$p_1 = 10$ MPa $\qquad h_1 = 3625.3$ kJ/kg

$t_1 = 600°C \qquad s_1 = 6.9029$ kJ/(kg) (K)

At $p_2 = 0.2$ MPa

$v_{f2} = 0.0010605 \qquad h_{f2} = 504.70 \qquad s_{f2} = 1.5301$

$v_{fg2} = 0.88464 \qquad h_{fg2} = 2201.9 \qquad s_{fg2} = 5.5970$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.9029 - 1.5301}{5.5970} = 0.9599$$

$h_2 = h_{f2} + x_2h_{fg2} = 504.70 + (0.9599)(2201.9) = 2618.3$ kJ/kg

$v_2 = v_{f2} + x_2v_{fg2} = 0.0010605 + (0.9599)(0.88464)$

$= 0.8502$ m³/kg

(a) $W = h_1 - h_2 = 3625.3 - 2618.3 = 1007$ kJ/kg

$$m = \frac{3600}{W} = \frac{3600}{1007} = 3.575 \text{ kg/kwh}$$

$$p_m = \frac{W}{V_D} = \frac{W}{v_2} = \frac{1007}{0.8502} = 1184.4 \text{kPa}$$

(b) $W_B = (n_b)(W) = (0.84)(1007) = 845.9$ kJ/kg

$h_{2'} = h_1 - W_B = 3625.3 - 845.9 = 2779.4$ kJ/kg

At $p_{2'} = 0.2$ MPa

| | | | | | |
|---|---|---|---|---|---|
| | | 150 | 2768.8 | | |
| 10 | x | $t_{2'}$ | 2779.4 | 10.6 | 20.5 |
| | | 160 | 2789.3 | | |

$$\frac{x}{10} = \frac{10.6}{20.5}$$

$x = 5.2$

$t_{2'} = 150 + 5.2 = 155.2°C$

$$W_K = (W_B)(\text{generator efficiency}) = (845.9)(0.93) = 786.7 \text{ kJ/kg}$$

$$m_k = \frac{3600}{W_K} = \frac{3600}{786.7} = 4.576 \text{ kg/kwh}$$

$$\text{Combined heat rate} = (m_k)(h_1 - h_{f2}) = (4.576)(3625.3 - 504.70)$$

$$= 14{,}280 \text{ kJ/kwh}$$

$$\text{Mass flow rate} = (30{,}000)(4.576) = 137{,}280 \text{ kg/h}$$

## The Reheat Cycle

### Introduction

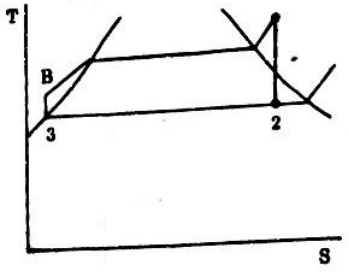


Fig. 3-5. Rankine Cycle

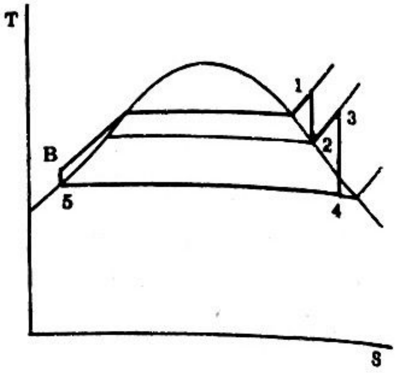


Fig. 3-6. Reheat Cycle

Moisture is harmful to the blades of the turbine. It causes erosion and cavitation of the turbine blades. As have been observed in the previous cycle (see Fig. 3-5), the moisture content increases during the later stages of the expansion process. One solution to this problem is by reheating the steam after partial expansion in the turbine (see Fig. 3-6). Reheating minimizes the moisture content and at the same time increases the efficiency of the cycle. Steam is usually withdrawn and reheated few degrees before the saturation point.

## The Ideal Reheat Cycle

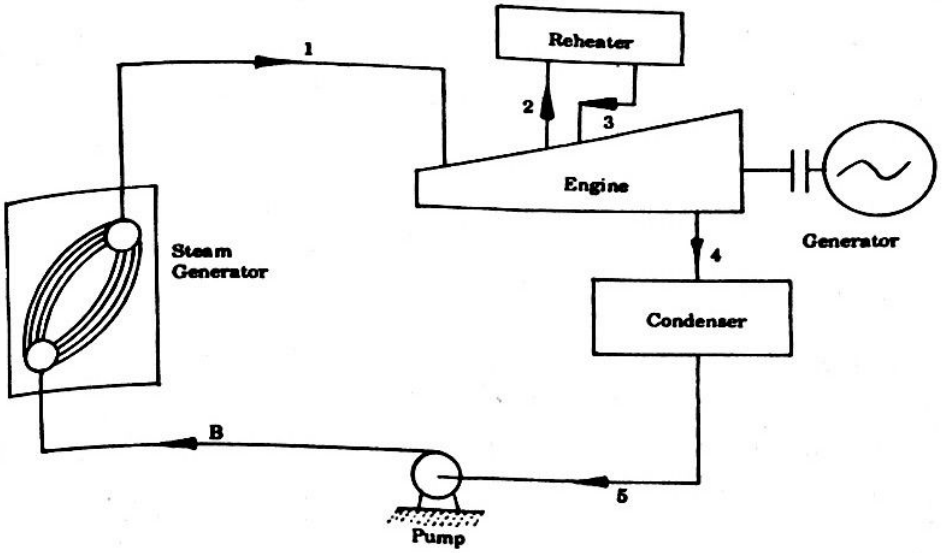


Fig. 3-7. Diagrammatic Layout for Reheating Plant

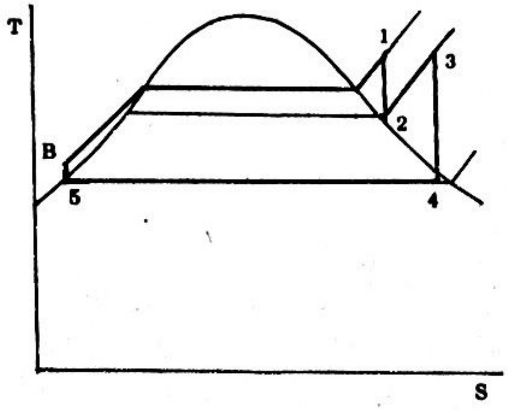


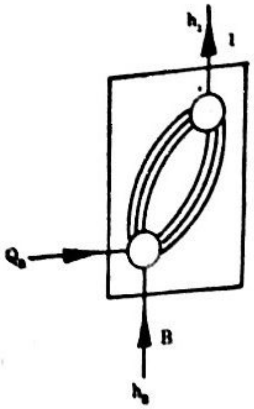
Fig. 3-8. Reheat Cycle With One stage of Reheating

The ideal reheat cycle with one stage of reheating is composed of the following processes:

1-2: partial isentropic expansion in the turbine, $s = C$
2-3: constant pressure resuperheating in the reheater, $p = C$
3-4: complete isentropic expansion in the turbine, $s = C$
4-5: constant pressure rejection of heat in the condenser, $p = C$
5-B: adiabatic pumping process, $s = C$
B-1: constant pressure addition of heat in the boiler, $p = C$

## Cycle Analysis

### A. Heat Added, $Q_A$



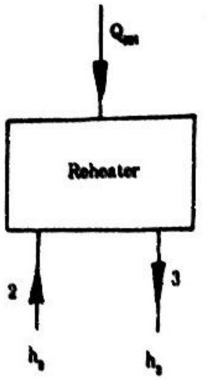
Energy balance:

$$E_{in} = E_{out}$$

$$Q_B + h_B = h_1$$

$$Q_B = h_1 - h_B$$

### Steam Generator



Energy balance:

$$E_{in} = E_{out}$$

$$Q_{RH} + h_2 = h_3$$

$$Q_{RH} = h_3 - h_2$$

$$Q_A = Q_B + Q_{RH}$$

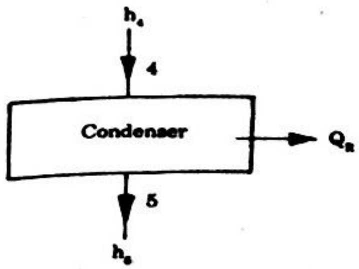
$$Q_A = h_1 - h_B + h_3 - h_2$$

For a given number of stages of reheating,

$$Q_A = Q_{Boiler} + \sum_1^n Q_{Reheater}$$

where: n = number of reheaters

### B. Heat Rejected, $Q_R$



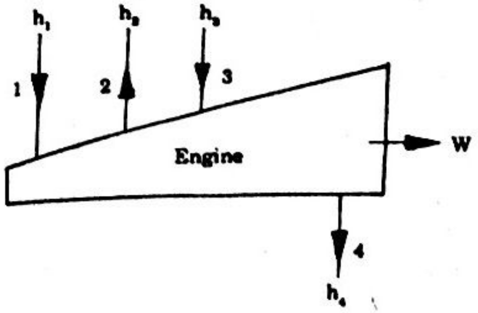
Energy balance:

$$E_{in} = E_{out}$$

$$h_4 = h_5 + Q_R$$

$$Q_R = h_4 - h_5$$

### C. Engine Work, W



Energy balance:

$$E_{in} = E_{out}$$

$$h_1 + h_3 = h_2 + h_4 + W$$

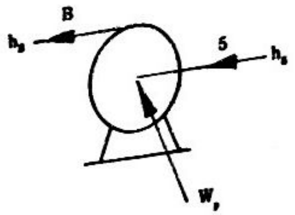
$$W = h_1 - h_2 + h_3 - h_4$$

Another means of determining engine work is by getting the sum of the work done by the steam during the different stages of expansion.

$$W = W_{1-2} + W_{3-4}$$

$$W = h_1 - h_2 + h_3 - h_4$$

### D. Pump Work, $W_p$



Approximate pump work

$$W_p \approx v_{f5}(p_B - p_5)$$

Exact pump work

$$E_{in} = E_{out}$$

$$W_p + h_5 = h_B$$

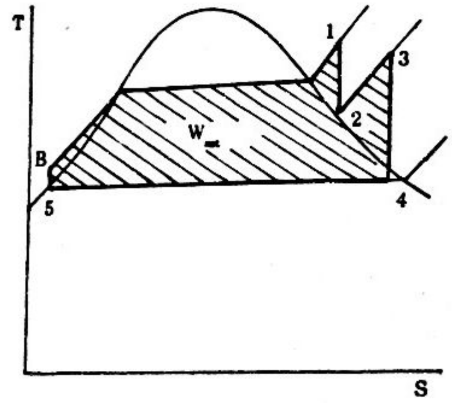
$$W_p = h_B - h_5$$

### E. Net Cycle Work, $W_{net}$

$$W_{net} = \text{Engine work} - \text{Pump work}$$

$$W_{net} = h_1 - h_2 + h_3 - h_4 - W_p$$

Another method:



$$W_{net} = Q_A - Q_R$$

$$= [h_1 - h_B + h_3 - h_2] - [h_4 - h_5]$$

$$= h_1 - h_2 + h_3 - h_4 - (h_B - h_5)$$

$$= h_1 - h_2 + h_3 - h_4 - W_p$$

### F. Thermal Efficiency, $e_e$

$$e_e = \frac{W_{net}}{Q_A}$$

$$W_{net} = h_1 - h_2 + h_3 - h_4 - W_p$$

$$Q_A = h_1 - h_B + h_3 - h_2$$

$$\text{but } h_B = h_5 + W_p$$

$$Q_A = h_1 - h_2 + h_3 - h_5 - W_p$$

$$e_e = \frac{h_1 - h_2 + h_3 - h_4 - W_p}{h_1 - h_2 + h_3 - h_5 - W_p}$$

### G. Steam Rate, $m_e$

$$m_e = \frac{3600}{W_{net}} \text{ kg/kwh}$$

## The Ideal Reheat Engine

The ideal reheat engine ignores the pressure drop in the reheater. The engine is an ideal one whether there is or there is no pressure drop in the reheater for as long as the expansion process is an isentropic one.

Engine Analysis

### A. Work, W

$$W = h_1 - h_2 + h_3 - h_4$$

### B. Steam Rate, $m_e$

$$m_e = \frac{3600}{W}$$

### C. Energy Chargeable Against the Engine, $E_e$

General equation which is applicable to a reheat engine only.

$$E_c = \text{enthalpy of steam entering the engine} - \text{enthalpy of saturated liquid at condensing temperature} + \Sigma Q_{RH}$$

For the given engine:

$$E_c = h_1 - h_5 + Q_{RH}$$
$$= h_1 - h_5 + (h_3 - h_2)$$
$$= h_1 - h_2 + h_3 - h_5$$

D. Thermal Efficiency, $e_e$

$$e_e = \frac{W}{E_c} = \frac{h_1 - h_2 + h_3 - h_4}{h_1 - h_2 + h_3 - h_5}$$

E. Heat Rate, HR

$$HR = m_e E_c$$

## The Actual Reheat Cycle

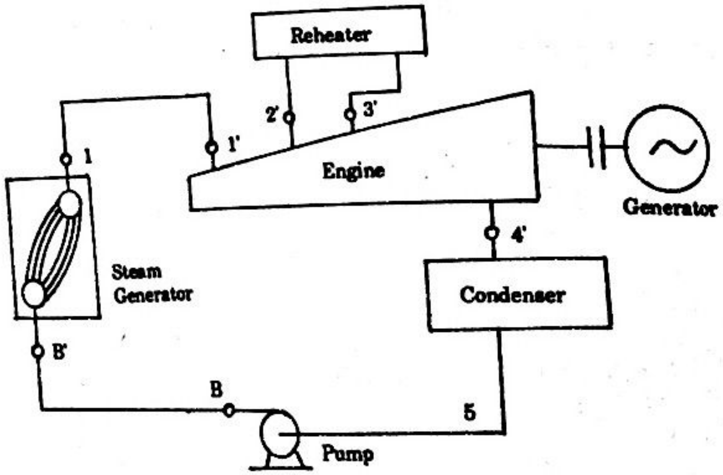


Fig. 3-9. Actual Reheat Cycle With One Stage of Reheating

Any presence of the following conditions will make the ideal reheat cycle an actual one.

a. **Pressure drop in the boiler/steam generator.**

$$p_{B'} \neq p_1 ; p_1 < p_{B'}$$

b. **Pressure drop in the steam line (1 – 1')**

$$p_{1'} < p_1 ; \; p_1 \neq p_{1'}$$
$$t_{1'} < t_1 ; \; t_1 \neq t_{1'}$$

c. **Pressure drop in the reheater.**

$$p_{2'} > p_{3'} ; p_{2'} \neq p_{3'}$$

d. **Pressure drop in the condenser.**

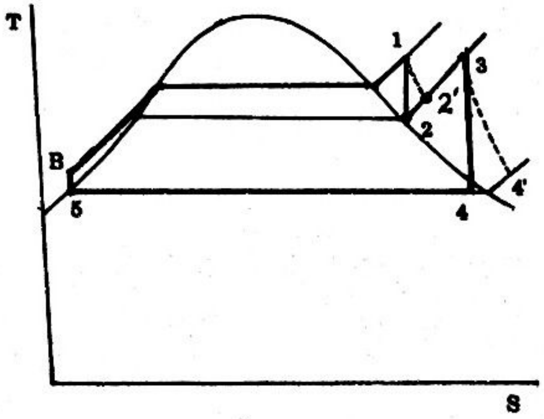
$$p_{4'} \neq p_5 ; p_5 < p_{4'}$$

e. **Irreversible adiabatic expansion process.**

$$s_{2'} \neq s_{1'} \text{ and } s_{4'} \neq s_{3'}$$

$$Q_{loss} = 0, \text{ but}$$

$$s_{2'} > s_{1'} \text{ and } s_{4'} > s_{3'}$$



General equation which is applicable to a reheat engine only.

$$E_c = \text{enthalpy of steam entering the engine} - \text{enthalpy of saturated liquid at condensing temperature} + \Sigma Q_{RH}$$

For the given engine:

$$E_c = h_1 - h_5 + Q_{RH}$$
$$= h_1 - h_5 + (h_3 - h_2)$$
$$= h_1 - h_2 + h_3 - h_5$$

D. Thermal Efficiency, $e_e$

$$e_e = \frac{W}{E_c} = \frac{h_1 - h_2 + h_3 - h_4}{h_1 - h_2 + h_3 - h_5}$$

E. Heat Rate, HR

$$HR = m_e E_c$$

## The Actual Reheat Cycle

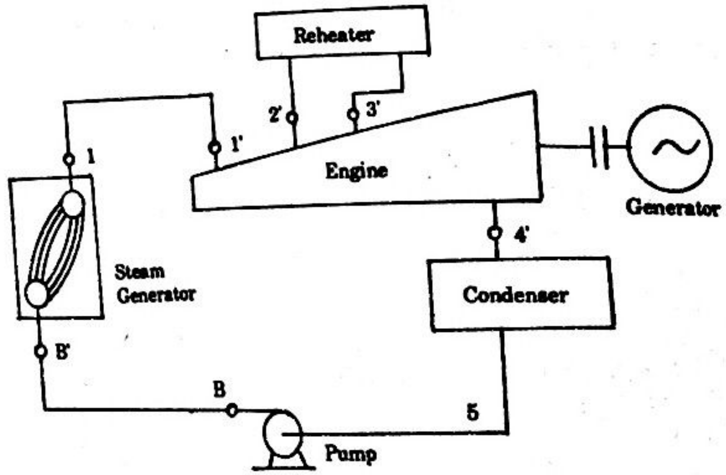


Fig. 3-9. Actual Reheat Cycle With One Stage of Reheating

Any presence of the following conditions will make the ideal reheat cycle an actual one.

a. Pressure drop in the boiler/steam generator.

$$p_{B'} \neq p_1 \,;\, p_1 < p_{B'}$$

b. Pressure drop in the steam line (1 – 1')

$$p_{1'} < p_1 ;\; p_1 \neq p_{1'}$$
$$t_{1'} < t_1 \,;\; t_1 \neq t_{1'}$$

c. Pressure drop in the reheater.

$$p_{2'} > p_{3'} \,;\, p_{2'} \neq p_{3'}$$

d. Pressure drop in the condenser.

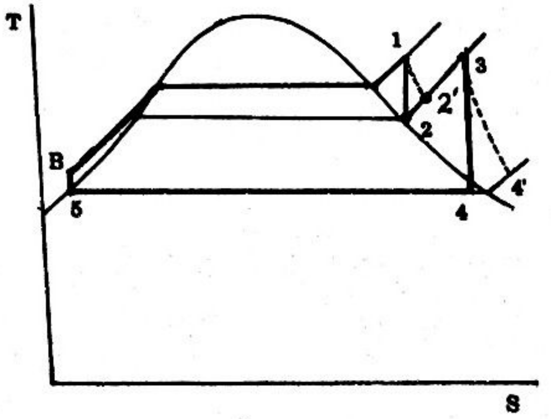
$$p_{4'} \neq p_5 \,;\, p_5 < p_{4'}$$

e. Irreversible adiabatic expansion process.

$$s_{2'} \neq s_{1'} \text{ and } s_{4'} \neq s_{3'}$$

$Q_{loss} = 0$, but

$$s_{2'} > s_{1'} \text{ and } s_{4'} > s_{3'}$$



C. Thermal Efficiency, $e_{e'}$

$$e_{e'} = \frac{W'}{E_{c'}}$$

D. Steam Rate, $m_{e'}$

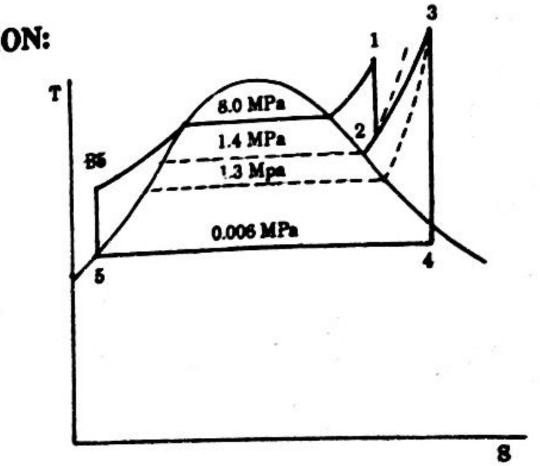
$$m_{e'} = \frac{3600}{W'}$$

E. Heat Rate, HR'

$$HR' = m_e E_{c'}$$

(3-5) In a reheat cycle steam at 8.0 MPa and 485°C enters the turbine and expands to 1.4 MPa. At this point, the steam is withdrawn and passed through a reheater. It re-enters the turbine at 1.3 MPa and 720°C. Expansion now occurs to the condenser pressure of 0.006 MPa. For the cycle and 1 kg of steam determine (a) $Q_A$, (b) $W_{net}$, and (c) $e_c$. For the engine determine (d) W, (e) $e_{c'}$, and (f) the steam flow for an engine output of 40,000 kW.

**SOLUTION:**



$h_1$ = h at 8.0 MPa and 485°C = 3361 kJ/kg
$h_2$ = h at 1.4 MPa and $s_2$ equal to $s_1$ = 2891 kJ/kg
$h_3$ = h at 1.3 MPa and 720°C = 3968 kJ/kg
$h_4$ = h at 0.006 MPa and $s_4$ equal to $s_3$ = 2526 kJ/kg
$h_5$ = $h_f$ at 0.006 MPa = 151.53 kJ/kg
$v_{f5}$ = $v_f$ at 0.006 MPa = 0.0010064 m³/kg
$W_p$ = $v_{f5}(p_{B5} - p_5)$ = 0.0010064 (8000 – 6) = 8.05 kJ/kg
$h_{B5}$ = $h_5 + W_p$ = 151.53 + 8.05 = 159.58 kJ/kg

(a) $Q_A = h_1 - h_{B5} + h_3 - h_2$
$= 3361 - 159.58 + 3968 - 2891$
$= 4278.4$ kJ/kg

(b) $W = h_1 - h_2 + h_3 - h_4$
$= 3361 - 2891 + 3968 - 2526$
$= 1912$ kJ/kg
$W_{net} = W - W_p = 1912 - 8.05 = 1904$ kJ/kg

(c) $e_c = \frac{W_{net}}{Q_A} = \frac{1904}{4278.4} = 0.445$ or 44.5%

(d) W = 1912 kJ/kg

(e) $E_c = h_1 - h_2 + h_3 - h_{f5}$
$= 3361 - 2891 + 3968 - 151.53$
$= 4286.5$ kJ/kg

$e_e = \frac{W}{E_c} = \frac{1912}{4286.5} = 0.446$ or 44.6%

(f) $m = \frac{3600}{W} = \frac{3600}{1912} = 1.88$ kg/kwh

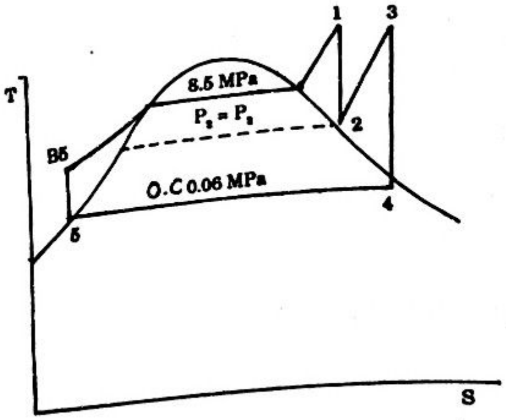
steam flow = (40,000 kw) (1.88 kg/kwh)
= 75,200 kg/h or 20.89 kg/s

(3-6) A reheat cycle has the following data:

Throttle, 8.5 MPa at 400°C
Turbine exhaust, 0.06 MPa

For an ideal cycle, determine the (a) work, (b) thermal efficiency, and (c) heat rate. (d) What is the thermal efficiency of the corresponding Rankine cycle, (e) What is the percent improvement in engine heat rate of the reheat cycle from the Rankine cycle.

**SOLUTION:**



$h_1$ = h at 8.5 MPa and 400°C = 3128.2 kJ/kg
$h_2$ = h at $s_2$ equal to $s_1$ and saturated = 2800 kJ/kg
$h_3$ = h at $p_3$ equal to $p_2$ and 400°C = 3246 kJ/kg
$h_4$ = h at 0.06 MPa and $s_4$ equal to $s_3$ = 2500 kJ/kg
$v_{f5}$ = $v_f$ at 0.06 MPa = 0.0010331 m³/kg
$W_p$ = $v_{f5}(p_1 - p_5) = 0.0010331\,(8{,}500 - 60) = 8.7$ kJ/kg
$h_{B5}$ = $h_5 + W_p = 359.86 + 8.7 = 368.6$ kJ/kg

(a) $W = h_1 - h_2 + h_3 - h_4$
$= 3128.2 - 2800 + 3246 - 2500$
$= 1074.2$ kJ/kg
$W_{net} = W - W_p = 1074.2 - 8.7 = 1065.5$ kJ/kg

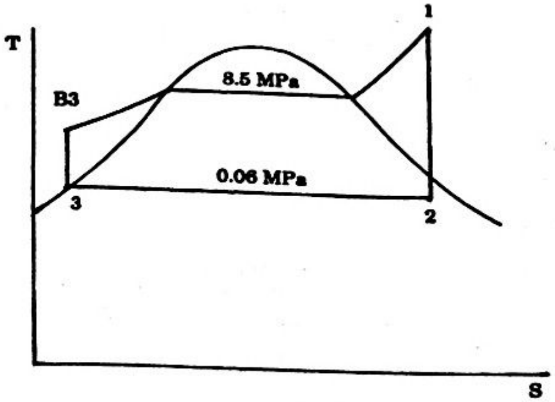
(b) $Q_A = h_1 - h_{B5} + h_3 - h_2$
$= 3128.2 - 368.6 + 3246 - 2800$
$= 3205.6$ kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{1065.5}{3205.6} = 0.3324 \text{ or } 33.24\%$$

(c) $$m = \frac{3600}{W} = \frac{3600}{1074.2} = 3.35 \text{ kg/kwh}$$

Heat rate = $m(h_1 - h_{f4}) = 3.35\,(3128\text{-}2 - 359.86)$
= 9273.9 kJ/kwh

(d) **The corresponding Rankine cycle**



$h_2$ = h at 0.06 MPa and $s_2$ equal to $s_1$ = 2220 kJ/kg
$h_3$ = $h_f$ at 0.06 MPa = 359.86 kJ/kg
$v_{f3}$ = $v_f$ at 0.06 MPa = 0.0010331 m³/kg
$W_p$ = $v_{f3}(p_1 - p_3) = 0.0010331\,(8{,}500 - 60) = 8.7$ kJ/kg
$h_{B3}$ = $h_3 + W_p = 359.86 + 8.7 = 368.6$ kJ/kg
$W = h_1 - h_2 = 3128.2 - 2220 = 908.2$ kJ/kg
$W_{net} = W - W_p = 908.2 - 8.7 = 899.5$ kJ/kg
$Q_A = h_1 - h_{B3} = 3128.2 - 368.6 = 2759.6$ kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{899.5}{2759.6} = 0.3259 \text{ or } 32.59\%$$

(e) Rankine engine steam rate $= \dfrac{3600}{W} = \dfrac{3600}{908.2}$
= 3.964 kg/kwh

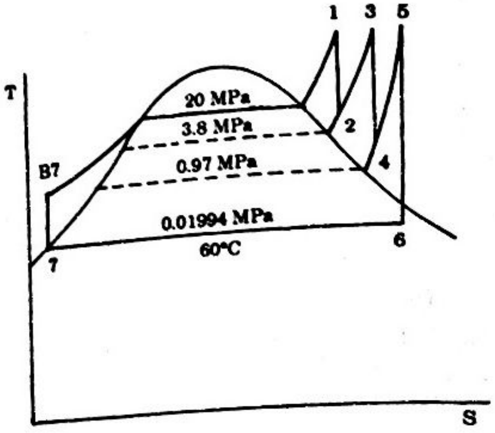
Rankine engine heat rate = $m(h_1 - h_{f2}) = (3.964)$
$(3128.2 - 359.86)$
= 10,974 kJ/kwh

Difference or reduction in heat rate = Rankine HR − Reheat HR = 10,974 − 9273.9 = 1700.1 kJ/kg

$$\% \text{ improvement} = \frac{1700.1}{10{,}974} = 0.1549 \text{ or } 15.49\%$$

(3-7) A reheat cycle with two stages of reheating is executed with steam expanding initially from 20 MPa and 540°C. The two reheater pressures are 3.8 MPa and 0.97 MPa, and the steam leaves each reheater at 540°C. Condensation occurs at 60°C. (a) For the ideal cycle and 1 kg/s of steam find $Q_A$ and $e_c$. (b) For the engine and 1 kg/s of steam find W and $e_e$.

**SOLUTION:**



$h_1$ = h at 20 MPa and 540°C = 3363.5 kJ/kg

$h_2$ = h at 3.8 MPa and $s_2$ equal to $s_1$ = 2916 kJ/kg

$h_3$ = h at 3.8 MPa and 540°C = 3538.8 kJ/kg

$h_4$ = h at 0.97 MPa and $s_4$ equal to $s_3$ = 3100 kJ/kg

$h_5$ = h at 0.97 MPa and 540°C = 3565.9 kJ/kg

$h_6$ = h at 0.01994 MPa and $s_6$ equal to $s_5$ = 2604 kJ/kg

$h_7$ = $h_f$ at 60°C = 251.13 kJ/kg

$v_{f7}$ = $v_f$ at 60°C = 0.0010172 m³/kg

$W_p = v_{f7}\,(p_{B7} - p_7) = 0.0010172\,(20{,}000 - 19.94)$

$= 20.32$ kJ/kg

$h_{B7} = h_7 + W_p = 251.13 + 20.32 = 271.45$ kJ/kg

(a) $Q_A = h_1 - h_{B7} + h_3 - h_2 + h_5 - h_4$

$= 3363.5 - 271.45 + 3538.8 - 2916 + 3565.9 - 3100$

$= 4180.8$ kJ/kg

$Q_A = (4180.8\ \text{kJ/kg})\,(1\ \text{kg/s})$

$= 4180.8$ kJ/s

$W = h_1 - h_2 + h_3 - h_4 + h_5 - h_6$

$= 3363.5 - 2916 + 3538.8 - 3100 + 3565.9 - 2604$

$= 1848.2$ kJ/kg

$W = (1848.2\ \text{kJ/kg})\,(1\ \text{kg/s})$

$= 1848.2$ kJ/s

$W_{net} = W - W_p = 1848.2 - 20.32 = 1827.9$ kJ/s

$$e_c = \frac{W_{net}}{Q_A} = \frac{1827.9}{4180.8} = 0.4337 \text{ or } 43.37\%$$

(b) $W = 1848.2$ kJ/s

$E_c = h_1 - h_7 + h_3 - h_2 + h_5 - h_4$

$= h_1 - h_2 + h_3 - h_4 + h_5 - h_7$

$= 3363.5 - 2916 + 3538.3 - 3100 + 3565.9 - 251.13$

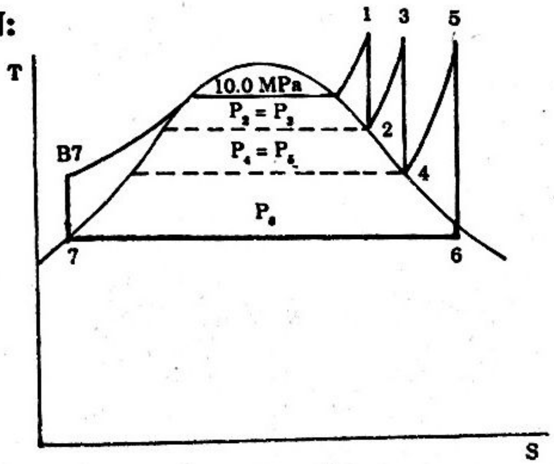
$= 4201.1$ kJ/kg

$E_c = (4201.1\ \text{kJ/kg})\,(1\ \text{kg/s}) = 4201.1$ kJ/s

$$e_e = \frac{W}{E_c} = \frac{1848.2}{4201.1} = 0.4399 \text{ or } 43.99\%$$

(3-8) An ideal double-reheat steam turbine receives 300,000 kg/h of steam at 10 MPa and 400°C. After expansion from high pressure turbine shell, it is reheated twice from saturated vapor condition back to its initial temperature. The steam enteringthe condenser is 97% dry. Determine (a) engine work, (b) engine thermal efficiency, and (c) steam rate. (d) Considering a generator output of 100 MW with 97% generator efficiency, what are the combined heat rate, brake work, brake thermal efficiency, and pump power requirement.

**SOLUTION:**



$h_1$ = h at 10 MPa and 400°C = 3096.5 kJ/kg
$h_2$ = h at $s_2$ equal to $s_1$ and saturated = 2804 kJ/kg
$h_3$ = h at $p_3$ equal to $p_2$ and 400°C = 3232 kJ/kg
$h_4$ = h at $s_4$ equal to $s_3$ and saturated = 2730 kJ/kg
$h_5$ = h at $p_5$ equal to $p_4$ and 400°C = 3274 kJ/kg
$h_6$ = h at $s_6$ equal to $s_5$ and 97% quality = 2510 kJ/kg
From Mollier Chart $p_6$ = 0.0086 MPa
$h_7$ = $h_f$ at 0.0086 MPa = 184.2 kJ/kg
$v_{f7}$ = $v_f$ at 0.0086 MPa = 0.0010095 m³/kg
$W_p$ = $v_{f7}(p_{B7} - p_7)$ = 0.0010095 (10,000 −8.6) =10.1 kJ/kg
$h_{B7}$ = $h_7 + W_p$ = 184.2 + 10.1 = 194.3 kJ/kg

$$\text{(a) } W = h_1 - h_2 + h_3 - h_4 + h_5 - h_6 = 3096.5 - 2804 + 3232 - 2730 + 3274 - 2510 = 1558.5 \text{ kJ/kg}$$

$$\text{(b) } E_c = h_1 - h_2 + h_3 - h_4 + h_5 - h_{f7} = 3096.5 - 2804 + 3232 - 2730 + 3274 - 184.2 = 3884.3 \text{ kJ/kg}$$

$$e_e = \frac{W}{E_c} = \frac{1558.5}{3884.3} = 0.4012 \text{ or } 40.12\%$$

(c) Steam rate, $m = \dfrac{3600}{W} = \dfrac{3600}{1558.5} = 2.31$ kg/kwh

(d) Combined steam rate, $m_k = \dfrac{300{,}000 \text{ kg/h}}{100{,}00 \text{ kw}} = 3.0$ kg/kwh

Combined heat rate = $(m_k)(E_c)$ = (3.0 kg/kwh) (3884.3 kJ/kg)
= 11,653 kJ/kwh

Combined work, $W_k = \dfrac{3600}{m_k} = \dfrac{3600}{3.0} = 1200$ kJ/kg

Brake work, $W_B = \dfrac{W_k}{\text{generator efficiency}} = \dfrac{1200}{0.97}$

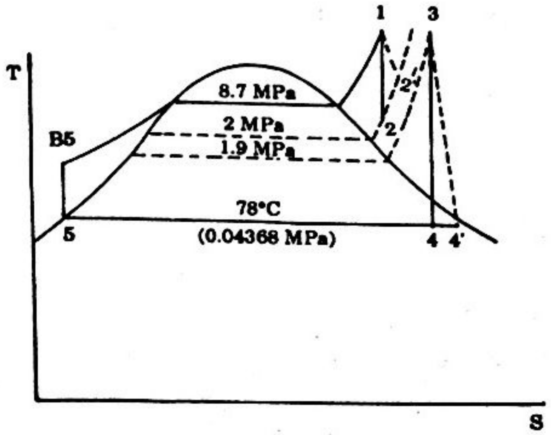
= 1237.1 kJ/kg

Brake thermal efficiency, $e_b = \dfrac{W_B}{E_c} = \dfrac{1237.1}{3884.3} = 0.3185$ or 31.85%

Pump power = $(m)\,W_p)$ = (300,000 kg/h) (10.1 kJ/kg)
= 3,030,000 kJ/h or 841.7 kw

**(3-9)** A steam turbine receives 362,500 kg/h of steam throttled at 8.70 MPa and 410°C. Steam leaves the high pressure turbine shell at 2 MPa with 7.58 degrees of superheat. Reheated steam is at 1.9 MPa and 410°C. Condensate is at 78°C. For a generator output of 100 MW at 94% efficiency, calculate (a) the combined thermal efficiency, (b) the actual condition of exhaust steam, (c) the thermal efficiency of the ideal engine, (d) the combined engine efficiency, and (e) the ideal thermal efficiency of the cycle without pressure drop through reheater.

**SOLUTION:**



$h_1$ = h at 8.70 MPa and 410°C = 3152.6 kJ/kg
$h_{2'}$ = h at 2 MPa and 220°C (212.42 + 7.58) = 2821.7 kJ/kg
$h_3$ = h at 1.90 MPa and 410°C = 3271.2 kJ/kg
$h_4$ = h at 0.04368 MPa ($p_{sat}$ at 78°C) and $s_4$ equal to $s_3$
= 2480 kJ/kg
$h_5$ = $h_f$ at 78°C = 326.51 kJ/kg
$v_{f5}$ = $v_f$ at 78°C = 0.0010278 m³/kg
$W_p$ = $v_{f5}(p_1 - p_5)$ = 0.0010278 (8,700 − 43.68) = 8.9 kJ/kg
$h_{B5}$ = $h_5 + W_p$ = 326.51 + 8.9 = 335.41 kJ/kg

(a) Combined steam rate, $m_k = \dfrac{362{,}500 \text{ kg/h}}{100{,}000 \text{ kw}} = 3.625$ kg/kwh

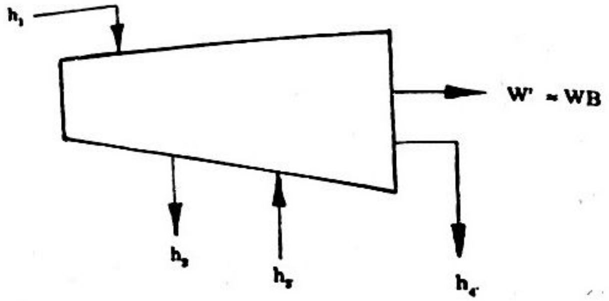
Combined work, $W_K = \frac{3600}{m_k} = \frac{3600}{3.625} = 993.1$ kJ/kg

$$\begin{aligned} E_c &= h_1 - h_{f4} + h_3 - h_{2'} \\ &= 3152.6 - 326.51 + 3271.2 - 2821.7 \\ &= 3275.6 \text{ kJ/kg} \end{aligned}$$

Combined thermal efficiency, $e_k = \frac{W_K}{E_c} = \frac{993.1}{3275.6}$

$= 0.3032$ or 30.32%

(b) Actual condition of exhaust steam



Energy balance:

$$\begin{aligned} h_1 + h_3 &= h_{2'} + h_{4'} + W' \\ h_{4'} &= h_1 + h_3 - h_2 - W' \end{aligned}$$

$$W' \approx W_B = \frac{W_K}{n_g} = \frac{993.1}{0.94} = 1056.5 \text{ kJ/kg}$$

$$\begin{aligned} h_{4'} &= 3162.6 + 3271.2 - 2821.7 - 1056.5 \\ &= 2545.6 \text{ kJ/kg} \\ h_{4'} &= h_{f4'} + x_{4'} h_{fg4'} \\ 2545.6 &= 326.51 + x_{4'} (2313.8) \\ x_{4'} &= 0.959 \text{ or } 95.9\% \end{aligned}$$

(c) For the ideal engine

$$\begin{aligned} h_2 &= \text{h at 2 MPa and } s_2 \text{ equal to } s_1 = 2804 \text{ kJ/kg} \\ W &= h_1 - h_2 + h_3 - h_4 \\ &= 3152.6 - 2804 + 3271.4 - 2840 \\ &= 1140 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} E_c &= h_1 - h_2 + h_3 - h_{f4} \\ &= 3152.6 - 2804 + 3271.4 - 326.51 \\ &= 3293.5 \text{ kJ/kg} \end{aligned}$$

$$e_e = \frac{W}{E_c} = \frac{1140}{3293.5} = 0.3461 \text{ or } 34.61\%$$

(d) Combined engine efficiency, $n_k = \frac{e_k}{e_e} = \frac{30.32}{34.61} = 0.8760$ or 87.60%

(e) Without pressure drop through reheater

$$\begin{aligned} h_3 &= \text{h at 2 MPa and 410°C} = 3269.6 \text{ kJ/kg} \\ h_4 &= \text{h at 0.04368 MPa and } s_4 \text{ equal to } s_3 = 2472 \text{ kJ/kg} \\ W_{net} &= h_1 - h_2 + h_3 - h_4 - W_p \\ &= 3152.6 - 2804 + 3269.6 - 2472 - 8.9 \\ &= 1137.3 \text{ kJ/kg} \\ Q_A &= h_1 - h_{B5} + h_3 - h_2 \\ &= 3152.6 - 335.41 + 3269.6 - 2804 \\ &= 3282.8 \text{ kJ/kg} \end{aligned}$$

$$e_c = \frac{W_{net}}{Q_A} = \frac{1137.3}{3282.8} = 0.3464 \text{ or } 34.64\%$$

## The Regenerative Cycle

### Introduction

The thermal efficiency of a simple steam power plant is less than fifty percent. This means that more than half of the heat added to the water in the boiler is just wasted and rejected in the condenser. In order to utilize some of these heats that would have been wasted and rejected in the condenser, part of the throttle steam is extracted or bled for feedwater heating after it has partially expanded in the turbine. The extraction/bled points occur near the saturation state. The process of heating feedwater in this manner is called regeneration and the cycle governing it is the REGENERATIVE CYCLE.

## Effects of Regenerative Feedwater Heating

1. Increase in thermal efficiency.

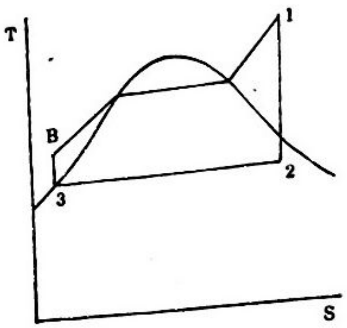


Fig. 3-10. Rankine Cycle

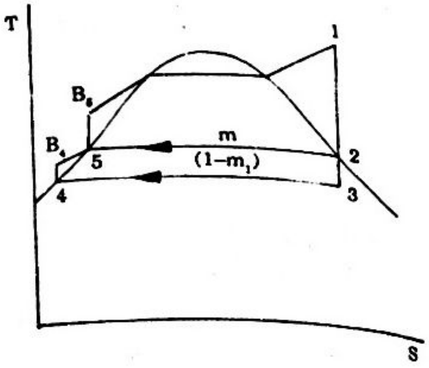


Fig. 3-11. Regenerative Cycle with One Stage Extraction for Feedwater Heating

By definition, $e = W_{net}/Q$. Examining the equation, the two ways of increasing the thermal efficiency are (a) by increasing the net cycle work and (b) by reducing the heat supplied, $Q_A$, (Refer to Fig. 3-10 and Fig. 3-11). The temperature of feed water entering the boiler in the regenerative cycle ($t_{B5}$) is higher than that of the original Rankine cycle ($t_B$). Since the feedwater enters the boiler at a relative high temperature, a smaller quantity of heat is needed to transform it to steam than without the regenerative feedwater heating. This in effect tend to increase the thermal efficiency.

It is true that the net work done per kilogram of the throttle steam in the regenerative cycle is less than that of the Rankine cycle as the consequence of the extraction of steam for feedwater heating. This tends to decrease the thermal efficiency. But the rate of decreased in the heat supplied, $Q_A$, is faster than the reduction rate in the net cycle work, $W_{net}$. Therefore, the net result of this is an increase in thermal efficiency.

2. Decrease in the moisture content during the later stages of expansion.

Refer to Fig. 3-10 and Fig. 3-11. It is a fact that the quality of exhaust steam for both cycles are the same, i.e., $x_2$ (Rankine cycle) = $x_3$ (Regenerative cycle). But the quantity of exhaust steam decreases in the regenerative cycle as the result of the bleeding process. Therefore, the moisture content decreases.

## The Ideal Regenerative Cycle With One Stage of Extraction for Feedwater Heating.

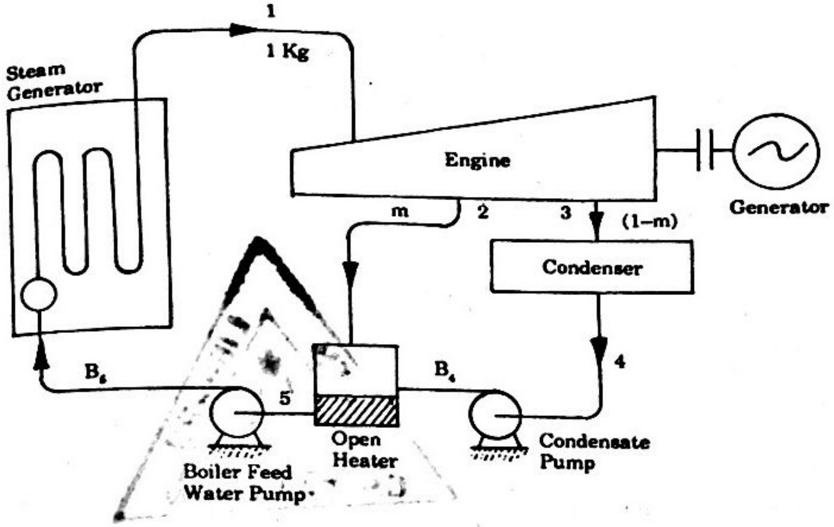


Fig. 3-12. Plant Layout of Regenerative Cycle With One Stage of Extraction for Feedwater Heating.

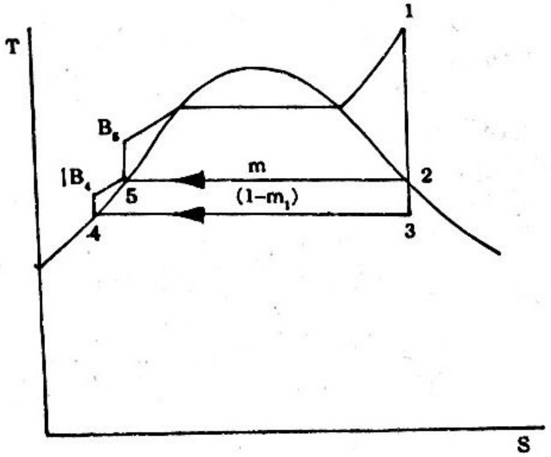
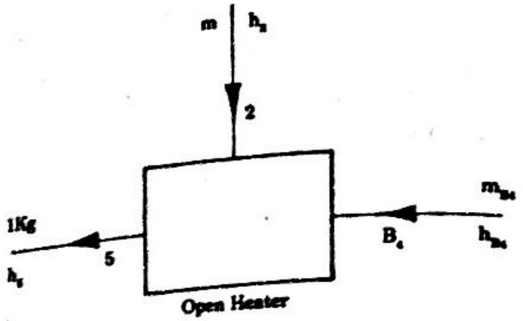


Fig. 3-11. Repeated

Cycle Analysis:
Basis: 1 kg of throttle steam

A. Mass of Bled Steam, m


m
$h_2$
2
$m_{B4}$
$h_{B4}$
$B_4$
1Kg
5
$h_5$
Open Heater

CITY LIBRARY OF TANAUAN

Mass balance:

$$m_{in} = m_{out}$$

$$m_{B4} + m = 1$$

$$m_{B4} = 1 - m$$

Energy balance:

$$E_{in} = E_{out}$$

$$mh_2 + m_{B4}h_{B4} = m_5h_5$$

$$mh_2 + (1-m)\,h_{B4} = (1)\,h_5$$

$$m = \frac{h_5 - h_{B4}}{h_2 - h_{B4}}$$

Alternate method:
Heat Balance:

Heat from bled steam = Heat to feedwater

$$m\,(h_2 - h_5) = m_{B4}\,(h_5 - h_{B4})$$

$$m\,(h_2 - h_5) = (1 - m)\,(h_5 - h_{B4})$$

$$m = \frac{h_5 - h_{B4}}{h_2 - h_{B4}}$$

The condensate pump work is often small so that it can be neglected. Neglecting condensate pump work,

$$h_{B4} \approx h_4$$

$$m = \frac{h_5 - h_4}{h_2 - h_4}$$

It can now be said that for any feedwater heater using direct contact type (open heaters),

$$\text{mass of bled steam} = [\text{mass of feedwater leaving the heater}] \left[ \frac{\text{enthalpy of feedwater leaving the heater} - \text{enthalpy of feedwater entering the heater}}{\text{enthalpy of bled steam} - \text{enthalpy of feedwater entering the heater}} \right]$$

(General Equation)

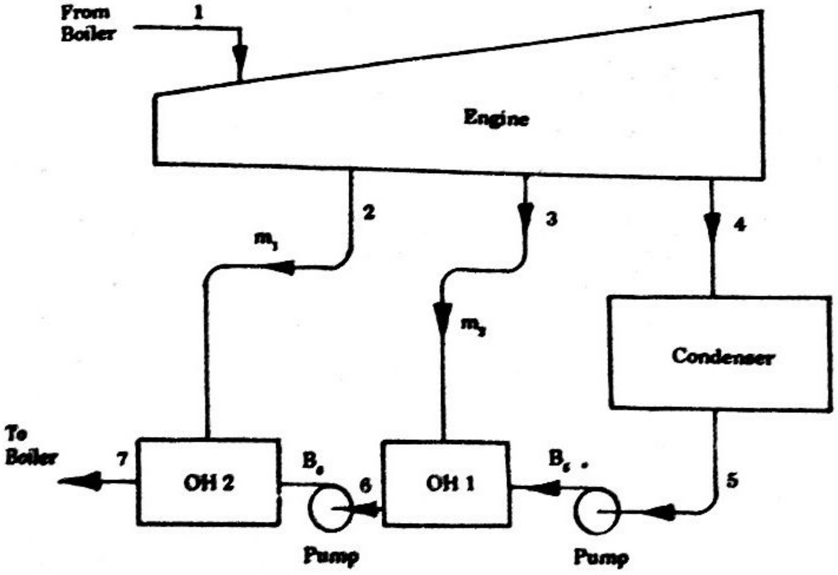

From Boiler
1
Engine
2
3
4
$m_1$
$m_2$
Condenser
To Boiler
7
OH 2
$B_6$
6
OH 1
$B_5$
5
Pump
Pump

Fig. 3.13 Regenerative Cycle With Two-Stages of Extraction For Feedwater Heating

Neglecting pump works,

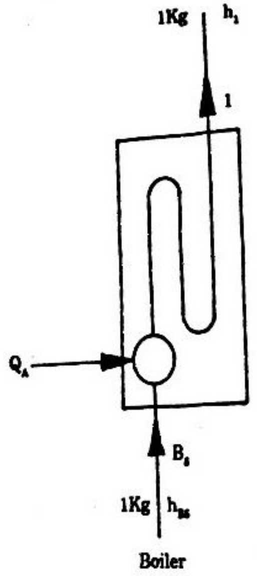
$$h_6 \approx h_{B6}$$
$$h_5 \approx h_{B5}$$

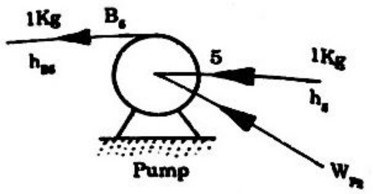
Applying the general equation for determining the quantity of bled steam,

$$m_1 = \frac{h_7 - h_6}{h_2 - h_6}$$

$$m_2 = (1 - m_1)\,\frac{h_6 - h_5}{h_3 - h_5}$$

## B. Heat Supplied, $Q_A$


1Kg
h₁
1
Q_A
B₅
1Kg
h_B5
Boiler


1Kg
B₅
h_B5
5
1Kg
h₅
Pump
W_p2

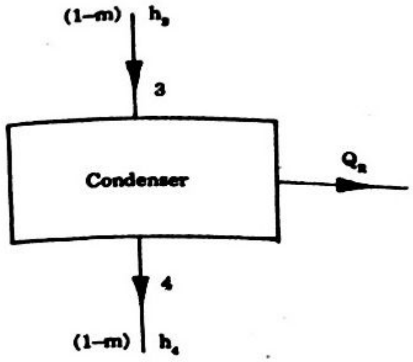
Energy balance:

$$E_{in} = E_{out}$$
$$W_{p2} + h_5 = h_{B5}$$
$$\text{Where: } W_{p2} = v_{f5}(p_{B5} - p_5)$$

Energy balance:

$$E_{in} = E_{out}$$
$$Q_A + h_{B5} = E_{out}$$
$$Q_A = h_1 - h_{B5}$$
$$\text{but } \quad h_{B5} = h_5 + W_{p2}$$
$$\text{Therefore, } \quad Q_A = h_1 - h_5 - W_{p2}$$

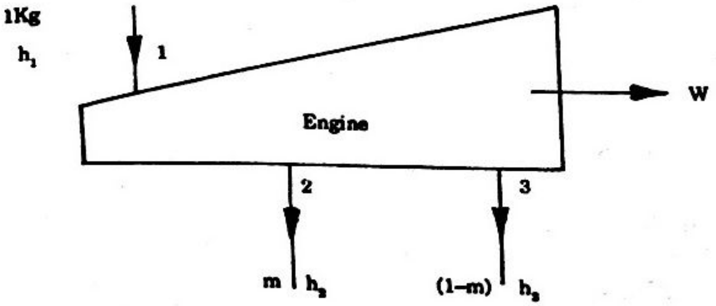
## C. Heat Rejected, $Q_R$


(1–m)
h₃
3
Condenser
Q_R
4
(1–m)
h₄

Energy balance:

$$E_{in} = E_{out}$$
$$(1-m)h_3 = Q_R + (1-m)h_4$$
$$Q_R = (1-m)(h_3 - h_4)$$

## D. Engine Work, W


1Kg
h₁
1
Engine
W
2
3
m
h₂
(1–m)
h₃

Energy balance:

$$E_{in} = E_{out}$$
$$h_1 = mh_2 + (1-m)\,h_3 + W$$
$$W = h_1 - mh_2 - (1-m)\,h_3$$
$$W = h_1 - mh_2 - (1-m)\,h_3 + h_2 - h_2$$
$$= (h_1 - h_2) + (1-m)\,h_2 - (1-m)\,h_3$$
$$W = (h_1 - h_2) + (1-m)(h_2 - h_3)$$

Another method:

$$W = \Sigma W \text{ (stage work)}$$
$$= W_{1\text{-}2} + W_{2\text{-}3}$$
$$= (h_1 - h_2) + (1-m)(h_2 - h_3)$$

E. Total Pump Work, $\Sigma W_p$

$$\Sigma W_p = W_{p1} + W_{p2}$$

Where: $W_{p1} = v_{f4}(p_{B4} - p_4)$
$W_{p2} = v_{f5}(p_{B5} - p_5)$

Approximate total pump work

$$\Sigma W_p = v_{f4}(p_{B5} - p_4)$$

F. Net Cycle Work, $W_{net}$

$$W_{net} = W - \Sigma W_p$$
$$= (h_1 - h_2) + (1 - m)(h_2 - h_3) - \Sigma W_p$$

G. Thermal Efficiency, $e_c$

$$e_c = \frac{W_{net}}{Q_A} = \frac{(h_1 - h_2) + (1 - m)(h_2 - h_3) - \Sigma W_p}{h_1 - h_5 - W_{p2}}$$

## The Ideal Regenerative Engine

Engine Analysis

A. Workk, W

$$W = (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

B. Energy Chargeable, $E_c$

The engine is charged with the enthalpy of steam entering the engine and credited with the enthalpy of feedwater leaving the last heater assuming that all the bled steam are used for feedwater heating.

$E_c$ = Enthalpy of steam entering the turbine — Enthalpy of feedwater leaving the last heater

**For the given cycle**

$$E_c = h_1 - h_5$$

**C. Thermal Efficiency, $e_e$**

$$e_e = \frac{W}{E_c}$$

## The Actual Regenerative Cycle

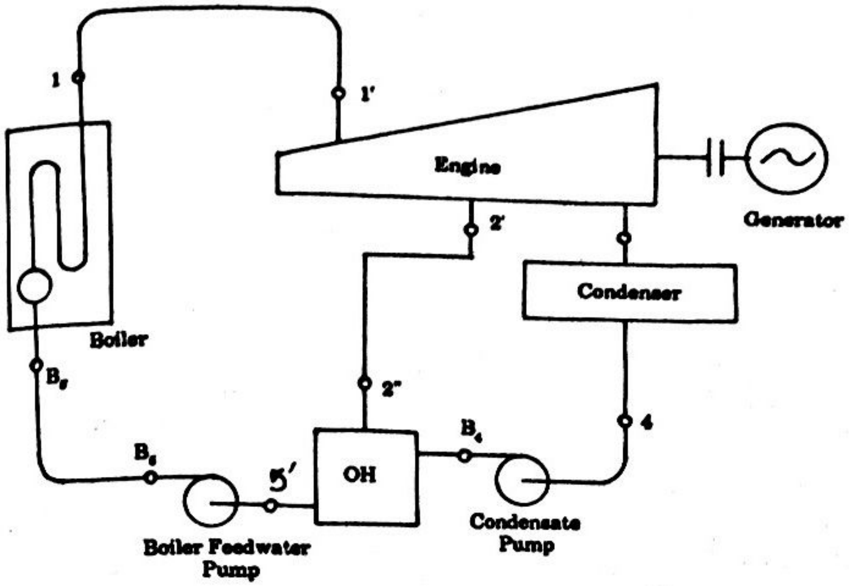


**Fig. 3-14. Plant Layout of Actual Regenerative Cycle with One Stage of Extraction for Feedwater Heating.**

**Any presence of the following conditions will make an ideal cycle an actual one.**

1. **Pressure drop in the boiler.**

$$p_1 < p_{B5'}$$

2. **Pressure drop in the steam line (1 – 1').**

$$p_{1'} < p_1$$

3. Pressure drop in the condenser.

$$p_4 < p_{3'}$$

4. Pressure drop in the bled steam line.

$$p_{2'} < p_{2}$$

5. Pressure drop in the feedwater line.

$$p_{B5'} < p_{B5}$$

6. Heat losses in the steam lines (1 – 1') and (2' – 2")

7. Heat losses in the turbine.
8. Inefficient pump
9. Heat losses in the heaters.

Cycle Analysis

A. Heat Added, $Q_{A'}$

$$Q_{A'} = h_1 - h_{B5'}$$

B. Heat Rejected, $Q_{R'}$

$$Q_{R'} = (1 - m')(h_{3'} - h_4)$$

C. Mass of Bled Steam

$$m' = \frac{h_5 - h_{B4}}{h_{2'} - h_{B4}} = \frac{h_{5'} - h_4}{h_{2'} - h_4}$$

D. Engine Work, W'

$$W' = (h_{1'} - h_{2'}) + (1 - m')(h_{2'} - h_{3'})$$

E. Pump Work, $W_{p'}$

$$W_{p'} = \Sigma W_p$$

$$= W_{p1'} + W_{p2'}$$

$$W_{p1'} = \frac{W_{p1}\text{(ideal)}}{\text{pump efficiency}}$$

$$W_{p2'} = \frac{W_{p2}\text{(ideal)}}{\text{pump efficiency}}$$

F. Net Cycle Work, $W_{net'}$

$$W_{net'} = W' - W_{p'}$$

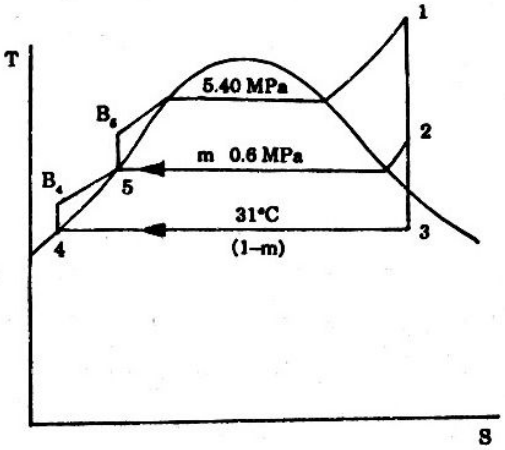
G. Thermal Efficiency, $e_{e'}$

$$e_{e'} = \frac{W' - W_{p'}}{Q_{A'}}$$

(3-10) Steam is delivered to an engine at 5.4 MPa and 600°C. Before condensation at 31°C, steam is extracted for feedwater heating at 0.60 MPa.

For an ideal cycle, find (a) the amount of steam extracted (b) W and (c) e. For an ideal engine and the same states, compute (d) W and e, and (e) steam rate.

**SOLUTION:**



$h_1$ = h at 5.40 MPa 600°C = 3663.3 kJ/kg
$h_2$ = h at 0.6 MPa and $s_2$ equal to $s_1$ = 2987 kJ/kg
$h_3$ = h at 0.004496 MPa and $s_3$ equal to $s_1$ = 2187 kJ/kg

$h_4 = h_f$ at 31°C = 129.97 kJ/kg

$h_5 = h_f$ at 0.60 MPa = 670.56 kJ/kg

$v_{f4} = v_f$ at 31°C = 0.0010046 m³/kg

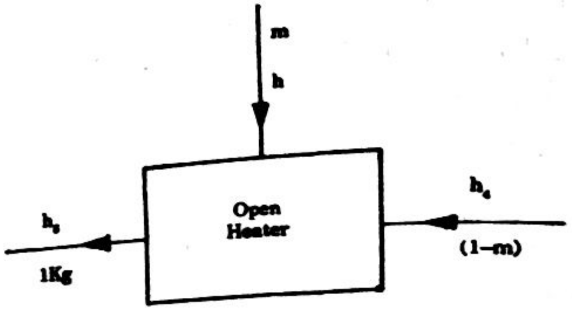
$v_{f5} = v_f$ at 0.60 MPa = 0.0011006 m³/kg

$W_{p2} = v_{f5}(P_{B5} - P_5) = (0.0011006)(5400 - 600) = 5.3$ kJ/kg

$\Sigma W_p = v_{f4}(P_{B5} - P_4) = (0.0010046)(5400 - 4.496)$

$= 5.42$ kJ/kg

$h_{B5} = h_5 + W_{p2} = 670.56 + 5.3 = 675.86$ kJ/kg



m
h
Open Heater
$h_4$
(1−m)
$h_5$
1Kg

Heat balance:

$$(1 - m)(h_5 - h_4) = m(h_2 - h_5)$$

$$m = \frac{h_5 - h_4}{h_2 - h_4} = \frac{670.56 - 129.97}{2978 - 129.97}$$

$$= 0.1898 \text{ kg/kg of throttle steam}$$

(b) $W_{net} = h_1 - h_2 + (1 - m)(h_2 - h_3) - \Sigma W_p$

$$= 3663.3 - 2978 + (1 - 0.1898)(2978 - 2187) - 5.42$$
$$= 1320.75 \text{ kJ/kg}$$

(c) $Q_A = h_1 - h_{B5} = 3663.3 - 675.86 = 2987.44$ kJ/kg

$$e_t = \frac{W_{net}}{Q_A} = \frac{1320.75}{2987.44} = 0.442 \text{ or } 44.2\%$$

(d) $W = h_1 - h_2 + (1 - m_1)(h_2 - h_3)$

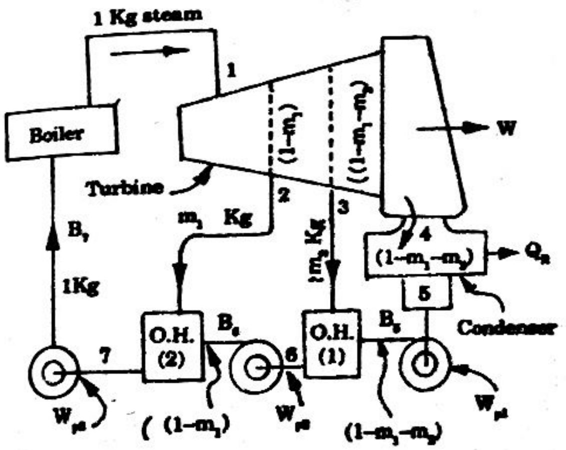
$$= 3663.3 - 2978 + (1 - 0.1898)(2978 - 2187)$$
$$= 1326.2 \text{ kJ/kg}$$

$$e_e = \frac{W}{h_1 - h_{f5}} = \frac{1326.2}{3663.3 - 670.56} = 0.443 \text{ or } 44.3\%$$

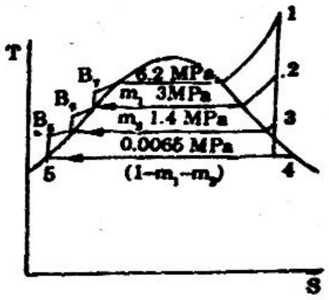
(e) $m = \dfrac{3600}{1326.2} = 2.71$ kg/kwh

(3-11) Steam at 6.2 MPa and 480°C is received by a regenerative engine. Extractions for feedwater heating occurs at 3 MPa and again at 1.4 MPa, with the remaining steam expanding to 0.0065 MPa. For the ideal engine with a throttle flow of 54,500 kg/h, find (a) the hourly quantities of steam extracted, (b) the turbine work in kW, and (c) the thermal efficiency. (d) For an ideal cycle through the same states, calculate $Q_A$ and $e_e$.

SOLUTION:



1 Kg steam
1
Boiler
Turbine
(1−m₁)
((1−m₁−m₂))
W
2
3
m₁ Kg
m₂ Kg
4
(1−m₁−m₂)
Q_R
5
Condenser
B₇
1Kg
O.H. (2)
B₆
O.H. (1)
B₅
7
6
W_p6
(1−m₁)
W_p5
(1−m₁−m₂)
W_p4



T
1
B₇
6.2 MPa
.2
B₆
m₁ 3MPa
B₅
m₂ 1.4 MPa
3
0.0065 MPa
5
(1−m₁−m₂)
4
S

$h_1$ = h at 6.2 MPa and 480°C = 3371.8 kJ/kg

$h_2$ = h at 3 MPa and $s_2$ equal to $s_1$ = 3150 kJ/kg

$h_3$ = h at 1.4 MPa and $s_3$ equal to $s_1$ = 2856 kJ/kg

$h_4$ = h at 0.0065 MPa and $s_4$ equal to $s_1$ = 2105 kJ/kg

$h_5$ = $h_f$ at 0.0065 MPa = 157.67 kJ/kg

$h_6$ = $h_f$ at 1.4 MPa = 830.30 kJ/kg

$h_7 = h_f$ at 3.0 MPa = 1008.42 kJ/kg

$v_{f5} = v_f$ at 0.0065 MPa = 0.0010069 m³ /kg

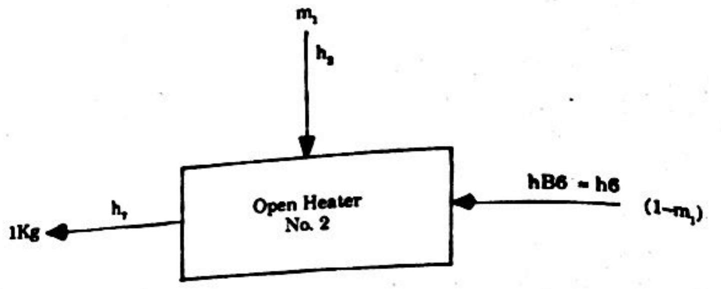
$v_{f7} = v_f$ at 3 MPa = 0.0012165 m³/kg

$W_{p3} = v_{f7}(p_{B7} - p_7) = (0.0012165)(3000 - 1400)$

$= 1.946$ kJ/k

$h_{B7} = h_7 + W_{p3} = 1008.42 + 1.946 = 1010.37$ kJ/kg

$\Sigma W_p = v_{f5}(p_{B7} - p_5) = (0.0010069)(3000 - 6.5) = 3.014$ kJ/kg



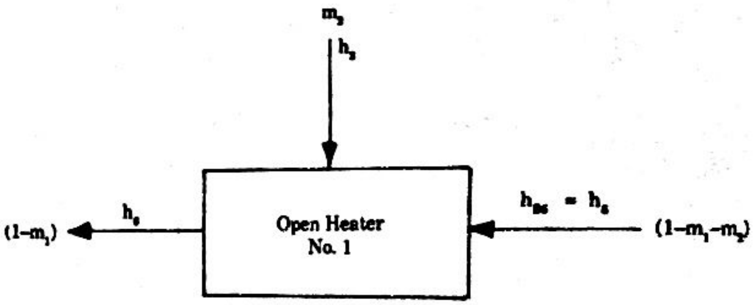
Heat balance:

$$m_1 h_2 + (1 - m_1)\, h_6 = h_7$$

$$m_1 = \frac{h_7 - h_6}{h_2 - h_6}$$

$$= \frac{1008.42 - 830.30}{3150 - 830.30}$$

= 0.0768 kg/kg throttle steam



Heat balance:

$$m_2 h_3 + (1 - m_1 - m_2)\, h_5 = (1 - m_1) h_6$$

$$m_2 = \frac{(1 - m_1)(h_6 - h_5)}{h_3 - h_5}$$

$$= \frac{(1-0.0768)(830.30-157.67)}{2856 - 157.67}$$

= 0.222 kg/kg throttle steam

(a) $m_1 = (0.0768)(54{,}500) = 4185.6$ kg/h

$m_2 = (0.222)(54{,}500) = 12{,}099$ kg/h

(b) $W = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$

$= (3371.8 - 3150) + (1 - 0.0768)(3150 - 2956)$

$+ (1 - 0.0768 - 0.222)(2956 - 2105)$

$= 997.62$ kJ/kg

$$W = (997.62)\left[\frac{54{,}500}{3600}\right] = 15{,}103 \text{ watts or } 15.102 \text{ kW}$$

(c) $E_c = h_1 - h_{f7} = 3371.8 - 1008.42 = 2363.38$ kJ/kg

$$e_e = \frac{W}{E_c} = \frac{997.62}{2363.38} = 0.4221 \text{ or } 42.21\%$$

(d) $Q_A = h_1 - h_{B7} = 3371.8 - 1010.37 = 2361.4$ kJ/kg

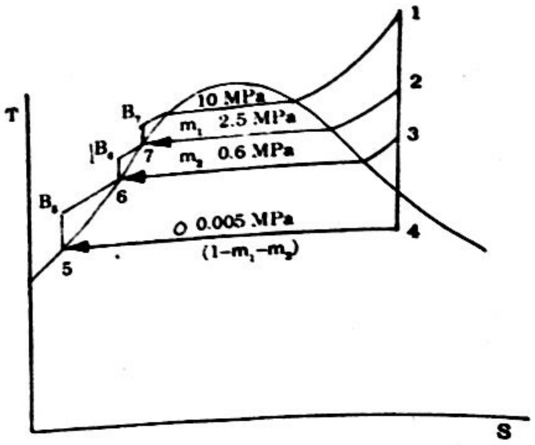
$$= (2361.4)\left[\frac{54{,}500}{3600}\right] = 35{,}750 \text{ watts or } 35.75 \text{ kW}$$

$W_{net} = W - \Sigma W_p = 997.62 - 3.014 - 994.61$ kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{994.61}{2361.4} = 0.4212 \text{ or } 42.12\%$$

**(3-12) An ideal regenerative cycle is executed with steam generation at 10.0 MPa and 600°C. Before condensation at 0.005 MPa, steam is extracted for feedwater heating first at 2.5 MPa and second at 0.6 MPa. On the basis of 1 kg of throttle steam, find (a) the quantity of steam extracted at each point, (b) $Q_A$ and $e_c$, (c) W, $e_e$, and m, and (d) for a combined steam rate of 3.34 kg/kwh, what is $n_k$?**

**SOLUTION:**



$h_1$ = h at 10.0 MPa and 600°C = 3625 kJ/kg
$h_2$ = h at 2.5 MPa and $s_2$ equal to $s_1$ = 3172 kJ/kg
$h_3$ = h at 0.6 MPa and $s_3$ equal to $s_1$ = 2820 kJ/kg
$h_4$ = h at 0.005 MPa and $s_4$ equal to $s_1$ = 2104 kJ/kg
$h_5$ = $h_f$ at 0.005 MPa = 137.82 kJ/kg
$h_6$ = $h_f$ at 0.6 MPa = 670.56 kJ/kg
$h_7$ = $h_f$ at 2.5 MPa = 962.11 kJ/kg
$v_{f5}$ = $v_f$ at 0.005 MPa = 0.0010053 m/kg
$v_{f7}$ = $v_f$ at 2.5 MPa = 0.0011973 m³/kg
$W_{p3} = v_{f7}(p_{B7} - p_7) = (0.0011973)(10{,}000 - 2500)$
= 8.98 kJ/kg
$h_{B7} = h_7 + W_{p3} = 962.11 + 8.98 = 971.09$ kJ/kg
$\Sigma W_p = v_{f5}(p_{B7} - p_5) = (0.0010063)(10{,}000 - 5)$
= 10.06 kJ/kg

(a) $(1 - m_1)(h_7 - h_6) = m_1(h_2 - h_7)$

$$m_1 = \frac{h_7 - h_6}{h_2 - h_6} = \frac{962.11 - 670.56}{3172 - 670.56}$$

$$= 0.11655 \text{ kg/kg}$$

$(1 - m_1 - m_2)(h_6 - h_5) = m_2(h_3 - h_6)$

$$m_2 = \frac{(1 - m_1)(h_6 - h_5)}{h_3 - h_5} = \frac{(1 - 0.11655)(670.56 - 137.82)}{2820 - 137.82}$$
$$= 0.17547 \text{ kg/kg}$$

(b) $Q_A = h_1 - h_{B7} = 3625 - 971.09 = 2653.91$ kJ/kg

$$W = h_1 - h_2 + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$$
$$= 3625 - 3172 + (1 - 0.11655)(3172 - 2820)$$
$$+ (1 - 0.11655 - 0.17547)(2820 - 2104)$$
$$= 1270.88 \text{ kJ/kg}$$

$W_{net} = W - \Sigma W_p = 1270.88 - 10.06 = 1260.82$ kJ/kg

$$e_c = \frac{W}{E_c} = \frac{1260.82}{2653.91} = 0.4751 \text{ or } 47.51\%$$

(c) W = 1270.88 kJ/kg

$E_c = h_1 - h_{f7} = 3625 - 962.11 = 2662.89$ kJ/kg

$$e_e = \frac{W}{E_c} = \frac{1270.88}{2662.89} = 0.4773 \text{ or } 47.73\%$$

$$m = \frac{3600}{m_k} = \frac{3600}{3.34} = 2.83 \text{ kg/kwh}$$

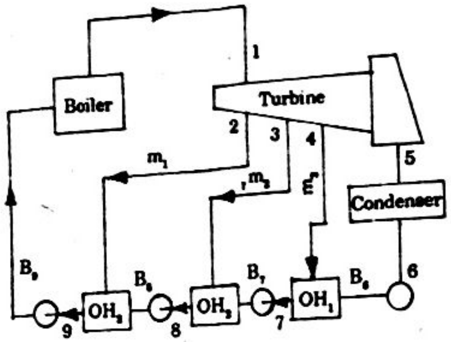
(d) $$W_k = \frac{2600}{m_k} = \frac{3600}{3.34} = 1077.84 \text{ kJ/kg}$$

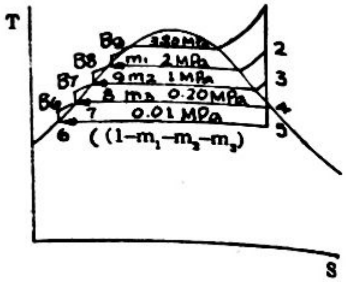
$$n_k = \frac{W_k}{W} = \frac{1077.84}{1270.88} = 0.8481 \text{ or } 84.81\%$$

**(3-13) A regenerative steam turbine has a throttle pressure of 3.8 MPa at 380°C and a condenser at 0.01 MPa. Steam are extracted at the following points: 2.0 MPa, 1.0 MPa and at 0.2 MPa. For the ideal cycle, find (a) thermal efficiency, steam rate**

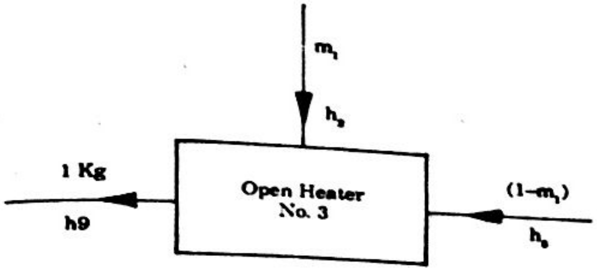
SR, and engine heat rate. (b) Estimate the percent improvement of heat rate from the corresponding Rankine engine. (c) If from actual test the combined steam rate is 5.63 kg/kwh and generator efficiency is 95%, calculate the actual amount of bled steam per kg of throttle flow using feedwater temperature leaving the last heater at 200°C. Assume fluid work is equal to brake work.

**SOLUTION:**


1
Boiler
Turbine
2
3
4
5
$m_1$
$m_2$
$m_3$
Condenser
$B_9$
$B_8$
$B_7$
$B_6$
6
$OH_3$
$OH_2$
$OH_1$
9
8
7


T
B9
380MPa
B8
m1 2MPa
B7
9 m2 1MPa
B6
8 m3 0.20MPa
7 0.01MPa
6 ((1–m₁–m₂–m₃)
2
3
4
5
S

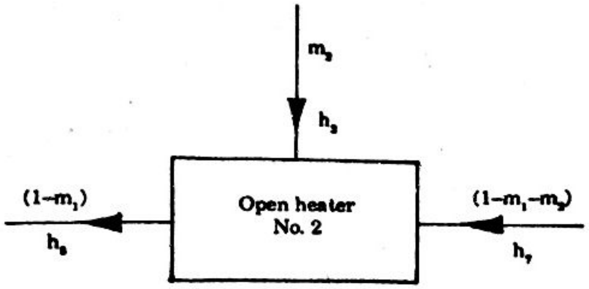
$h_1$ = h at 3.80 MPa, 380°C = 3169.8 kJ/kg
$h_2$ = h at 2.0 MPa and $s_2$ equal to $s_1$ = 3000 kJ/kg
$h_3$ = h at 1.0 MPa and $s_3$ equal to $s_1$ = 2843 kJ/kg
$h_4$ = h at 0.2 MPa and $s_4$ equal to $s_1$ = 2550 kJ/kg
$h_5$ = h at 0.01 MPa and $s_5$ equal to $s_1$ = 2131 kJ/kg
$h_6$ = $h_f$ at 0.01 MPa = 191.3 kJ/kg
$h_7$ = $h_f$ at 0.2 MPa = 504.7kJ/kg
$h_8$ = $h_f$ at 1.0 MPa = 762.81 kJ/kg
$h_9$ = $h_f$ at 2 MPa = 908.79 kJ/kg
$h_{f8}$ = $v_f$ at 0.01 MPa = 0.0010102 m³/kg
$v_{f9}$ = $v_f$ at 2 MPa = 0.0011767 m³/kg
$W_{p4} = v_{f9}(P_{B9} - P_9) = (0.0011767)(3800 - 2000)$
$= 2.12$ kJ/kg
$\Sigma W_p = v_{f8}(P_{B9} - P_6) = (0.0010102)(3800 - 10) = 3.83$ kJ/kg


$m_1$
$h_2$
1 Kg
h9
Open Heater No. 3
$(1-m_1)$
$h_8$

Heat balance:

$$(1 - m_1)(h_8 - h_9) = m_1(h_2 - h_9)$$

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8} = \frac{908.79 - 762.81}{3000 - 762.81} = 0.065 \text{ kg/kg of throttle steam}$$


$m_2$
$h_3$
$(1-m_1)$
$h_8$
Open heater No. 2
$(1-m_1-m_2)$
$h_7$

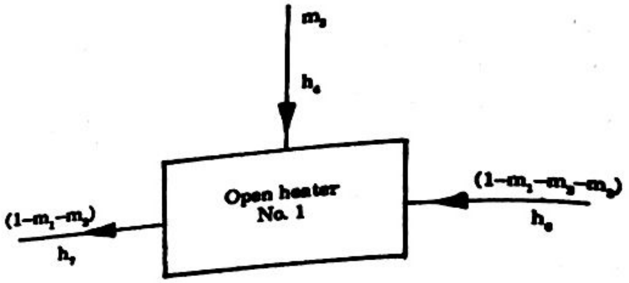
Heat balance:

$$(1 - m_1 - m_2)(h_8 - h_7) = m_2(h_3 - h_8)$$

$$m_2 = (1 - m_1)\left[\frac{h_8 - h_7}{h_3 - h_7}\right]$$

$$= (1 - 0.065)\left[\frac{762.81 - 504.7}{2843 - 504.7}\right]$$

$$= 0.103 \text{ kg/kg of throttle steam}$$


$m_3$
$h_4$
Open heater
No. 1
$(1-m_1-m_2-m_3)$
$h_8$
$(1-m_1-m_2)$
$h_7$

Heat balance:

$$(1 - m_1 - m_2 - m_3)(h_7 - h_8) = m_3 (h_4 - h_7)$$

$$m_3 = (1 - m_1 - m_2) \left[\frac{h_7 - h_8}{h_4 - h_8}\right]$$

$$= (1 - 0.065 - 0.103) \left[\frac{504.7 - 191.3}{2550 - 504.3}\right]$$

$$= 0.11 \text{ kg/kg of throttle steam}$$

$$W = h_1 - h_2 + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) + (1 - m_1 - m_2 - m_3)(h_4 - h_5)$$

$$= 3169.2 - 3000 + (1 - 0.065)(3000 - 2843) + (1 - 0.065 - 0.103)(2843 - 2550) + (1 - 0.065 - 0.103 - 0.11)(2550 - 2131)$$

$$= 862.89 \text{ KJ/kg}$$

$$W_{net} = W - \Sigma W_p = 862.89 - 3.83 = 859.06 \text{ kJ/kg}$$

$$Q_A = h_1 - h_9 - W_{p4}$$

$$= 3169.8 - 908.79 - 2.12 = 2258.88 \text{ kJ/kg}$$

(a) $$e_c = \frac{W_{net}}{Q_A} = \frac{859.06}{2258.88} = 0.38 \text{ or } 38\%$$

$$m = \frac{3600}{W} = \frac{3600}{862.89} = 4.172 \text{ kg/kwh}$$

$$HR = m(h_1 - h_6) = 4.172(3169.8 - 908.79)$$

$$= 9472.93 \text{ kJ/kwh}$$

(b) For Rankine Engine

$$W = h_1 - h_5 = 3169.8 - 2131 = 1038.8 \text{ kJ/kg}$$

$$E_c = h_1 - h_6 = 3169.8 - 191.3 = 2977.97 \text{ kJ/kg}$$

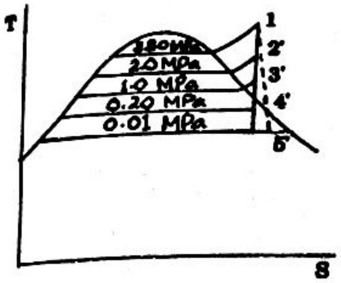
$$m = \frac{3600}{W} = \frac{3600}{1038.8} = 3.465 \text{ kg/kwh}$$

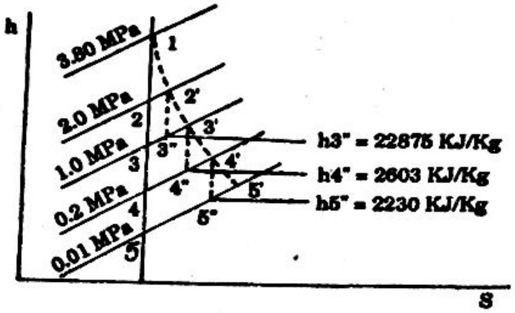
$$HR = (m) E_c) = 3.465 (2977.97) = 10320.5 \text{ kJ/kwh}$$

Heat rate difference = 10320.5 – 9432.93 = 887.57 kJ/kwh

$$\%\text{ improvement} = \frac{887.57}{10320.5} = 0.086 \text{ or } 8.6\%$$

(c) Actual Cycle


T
3.80 MPa
2.0 MPa
1.0 MPa
0.20 MPa
0.01 MPa
1
2'
3'
4'
5'
S


h
3.80 MPa
2.0 MPa
1.0 MPa
0.2 MPa
0.01 MPa
1
2
3
4
5
2'
3'
4'
5'
3"
4"
5"
h3" = 22875 KJ/Kg
h4" = 2603 KJ/Kg
h5" = 2230 KJ/Kg
S

For $m_k$ = 5.63 kg/kwh and $\eta_g$ = 95%

$$m_B = \eta_g m_k = 0.95 (5.63) = 5.3485 \text{ kg/kwh}$$

$$W_B = \frac{3600}{m_B} = \frac{3600}{5.3485} = 673 \text{ kJ/kg}$$

$$\eta_B = \frac{W_B}{W} = \frac{673}{862.89} = 0.78 \text{ or } 78\%$$

For $h_3$

$$0.78 = \frac{h_1 - h_3}{h_1 - h_{3'}} = \frac{3037.36 - h_3}{3037.36 - 2875}$$

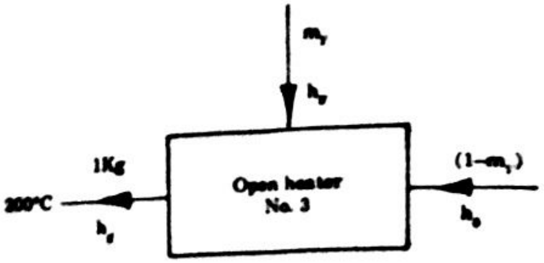
$$h_3 = 2910 \text{ kJ/kg}$$

For $h_4$

$$0.78 = \frac{h_3 - h_4}{h_3 - h_{4'}} = \frac{2910 - h_4}{2910 - 2603} = 2670.5 \text{ kJ/kg}$$

For $h_5$

$$0.78 = \frac{h_4 - h_5}{h_4 - h_{5'}} = \frac{2670.5 - h_5}{2670.5 - 2230} = 2326.8 \text{ kJ/kg}$$

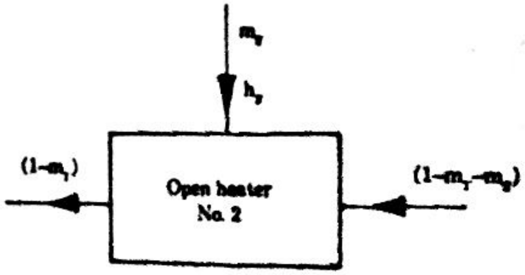


Heat balance:

$$h_9 = h_f \text{ at } 200°\text{C} = 852.45 \text{ kJ/kg}$$

$$(1 - m_1)(h_9 - h_8) = m_1 (h_2 - h_9)$$

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8} = \frac{852.45 - 762.81}{3037.36 - 762.81} = 0.03941 \text{ kg/kg of throttle steam}$$



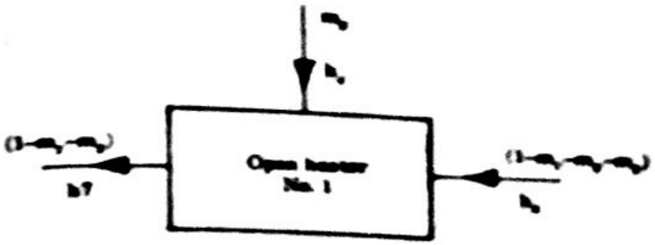
Heat balance:

$$(1 - m_1 - m_2)(h_8 - h_7) = m_2 (h_3 - h_8)$$

$$m_2 = (1 - m_1)\frac{h_8 - h_7}{h_3 - h_7}$$

$$= (1 - 0.03941)\frac{(762.81 - 504.7)}{(2910 - 504.7)}$$

$$= 0.013 \text{ kg/kg of throttle steam}$$



Heat balance:

$$(1 - m_1 - m_2 - m_3)(h_7 - h_6) = m_3 (h_4 - h_7)$$

$$m_3 = (1 - m_1 - m_2)\left[\frac{h_7 - h_6}{h_4 - h_7}\right]$$

$$= (1 - 0.03941 - 0.203)\left[\frac{504.7 - 191.3}{2670 - 191.3}\right]$$

$$= 0.1084 \text{ kg/kg of throttle steam}$$

## The Reheat-Regenerative Cycle

In this cycle, the reheat cycle and the regenerative cycle are combined to attain the following objectives.

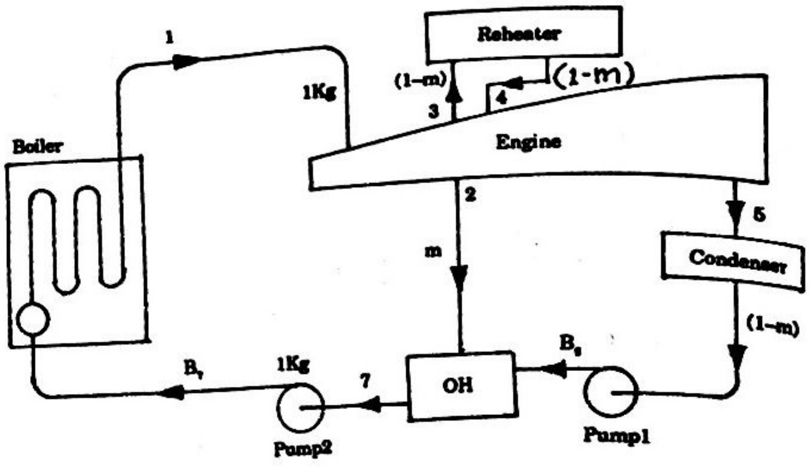
1. Further improvement in the overall thermal efficiency.
2. Further reduction in the moisture content of steam during the latter part of the expansion process.

### Cycle Analysis

The solution to a reheat regenerative cycle problem differs from that of the previous cycles namely: reheat cycle, Rankine cycle, and regenerative cycle. It does not follow fixed set of formulas nor there is a fixed pattern. Each problem requires a particular solution although the same laws, definitions, and principles are still being used. Problems can best be solved by energy balance and mass balance.

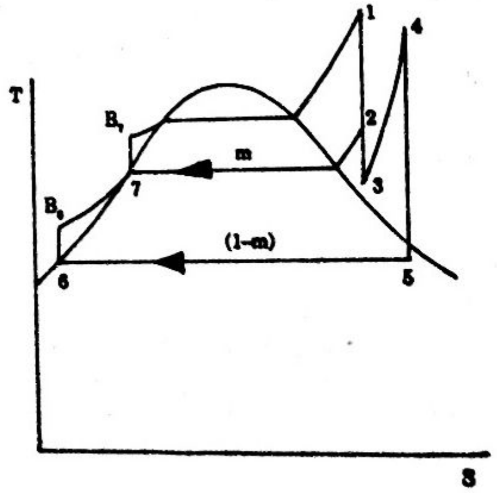
### CASE 1.

Assume an ideal reheat regenerative cycle: after some expansion, steam is extracted for feedwater heating; after further expansion, there is a reheat; then expansion to exhaust. Write the equations for (a) the quantity of extracted steam, (b) the net work, and (c) the thermal efficiency. The equations should refer to a Ts diagram with named points.



1
1Kg
Reheater
(1-m)
(1-m)
3
4
Boiler
Engine
2
m
5
Condenser
(1-m)
B6
B7
1Kg
7
OH
Pump2
Pump1

Fig. 3-15. Plant Layout of Case 1.



T
1
4
B7
2
m
7
3
B6
(1-m)
6
5
S

Fig. 3-15. Ts diagram of Case 1.

a. **Masss of Bled Steam, m**
**General Equation:**

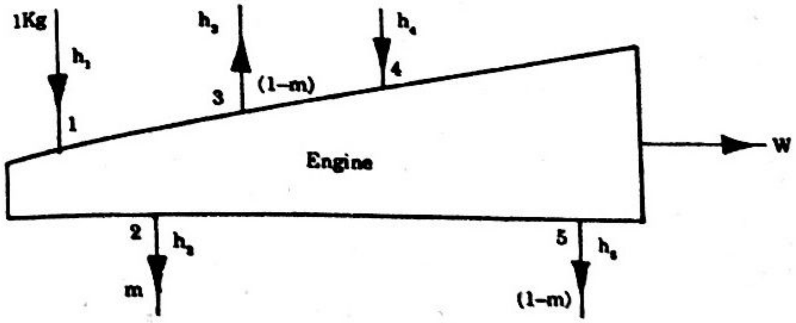
$$\text{mass of bled steam} = \left[\begin{array}{l}\text{mass of feedwater}\\ \text{leaving the heater}\end{array}\right]\left[\frac{\begin{array}{l}\text{enthalpy of feedwater}\\ \text{leaving the heater}\end{array} - \begin{array}{l}\text{enthalpy of feedwater}\\ \text{entering the heater}\end{array}}{\begin{array}{l}\text{enthalpy of bled}\\ \text{steam}\end{array} \cdot \begin{array}{l}\text{enthalpy of feedwater}\\ \text{entering the heater}\end{array}}\right]$$

$$m = (1)\left[\frac{h_7 - h_{B6}}{h_2 - h_{B6}}\right]$$

Neglecting condensate pump work (Pump 1), $h_{B6} \approx h_6$

$$m = \frac{h_7 - h_6}{h_2 - h_6}$$

b. **Net Cycle Work, $W_{net}$**
**Engine Work, W**



1Kg
$h_1$
$h_3$
$h_4$
1
3
(1-m)
4
Engine
W
2
$h_2$
m
5
$h_5$
(1-m)

First method

Energy balance:

$$E_{in} = E_{out}$$

$$h_1 + (1-m)h_4 = mh_2 + (1-m)h_3 + (1-m)h_5 + W$$

$$W = h_1 + (1-m)\,h_4 - mh_2 - (1-m)\,(h_3 + h_5)$$

Second method

$$W = \Sigma W \text{ of stages of expansion}$$

$$= W_{1-2} + W_{2-3} + W_{4-5}$$

$$= h_1 - h_2 + (1-m)(h_2 - h_3) + (1-m)(h_4 - h_5)$$

Total Pump Work, $\Sigma W_p$

$$\Sigma W_p = W_{p1} + W_{p2}$$

$$\Sigma W_p = v_{f8}(p_{B6} - p_6) + v_{f7}(P_{f7} - P_7)$$

(Exact formula)

$$\Sigma W_p = v_{f8}(p_{B7} - p_6)$$

(Approximate formula)

$$W_{net} = \text{Engine work} - \text{Pump work}$$

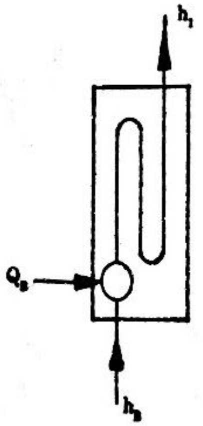
$$= W - \Sigma W_p$$

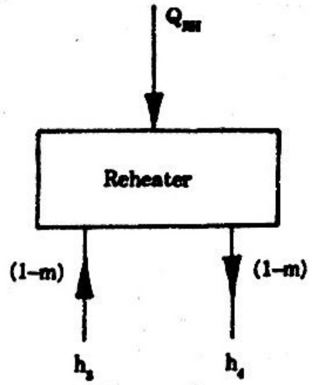
$$= (h_1 - h_2) + (1-m)(h_2 - h_3) + (1-m)(h_4 - h_5) - v_{f8}(p_{B7} - p_6)$$

c. Thermal Efficiency, e

$$e = \frac{W_{net}}{Q_A}$$

Heat Added $Q_A$





QRH
Reheater
(1–m)
(1–m)
h₃
h₄

$$Q_B = h_1 - h_{B7}$$

$$= h_1 - h_7 - W_{p2} \qquad Q_{RH} = (1-m)(h_4 - h_3)$$

$$W_{p2} = v_{f7}(p_{B7} - p_7)$$

General Equation:

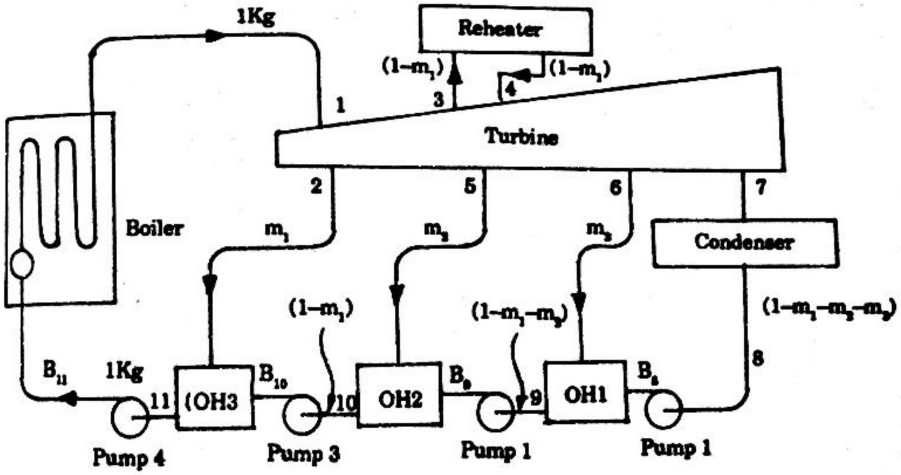
$$Q_A = Q_{Boiler} + \Sigma Q_{Reheaters}$$

$$= (h_1 - h_7 - W_{p2}) + (1-m)(h_4 - h_3)$$

$$e = \frac{(h_1 - h_2) + (1-m)(h_2 - h_3) + (1-m)(h_4 - h_5) - (v_{f8}(p_{B7} - p_6)}{[h_1 - h_7 - v_{f7}(p_{B7} - p_7)] + (1-m)(h_4 - h_3)}$$

CASE 2.

Assume an ideal reheat regenerative cycle with, first, an extraction for feedwater heating, then later a single reheating, and finally, two extraction points for feedwater heating. Sketch the energy diagram and write equations for (a) the quantity of steam extracted at each point, (b) the work from $Q_A$ and $Q_R$ and the turbine work, and (c) the thermal efficiency of the cycle. The equation should refer to a Ts diagram with named points.



1Kg
Reheater
(1–m₁)
(1–m₁)
1
3
4
Turbine
Boiler
2
5
6
7
m₁
m₂
m₃
Condenser
(1–m₁)
(1–m₁–m₂)
(1–m₁–m₂–m₃)
8
B₁₁
1Kg
11
(OH3
B₁₀
10
OH2
B₉
9
OH1
B₈
Pump 4
Pump 3
Pump 1
Pump 1

Fig. 3-16. Plant Layout of Case 2.

**Note:** Pump 1 is condenste pump
Pump 4 is main boiler feedwater pump

$p_2 = p_{11} = p_{B10}$
$p_5 = p_{10} = p_{B9}$
$p_6 = p_9 = p_{B8}$

$p_{B11} = p_1$
$p_7 = p_8$
$p_3 = p_4$

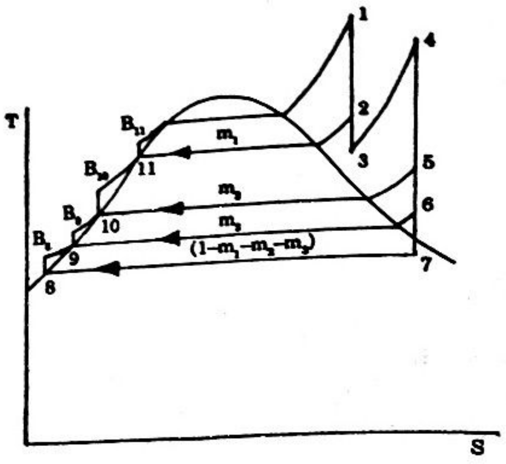

T
S
1
2
3
4
5
6
7
8
9
10
11
B11
B10
B9
B8
m1
m2
m3
(1–m1–m2–m3)

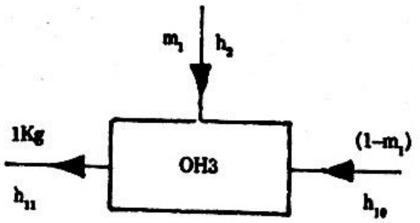
**Fig. 3-17. Ts diagram of Case 2.**

### a. Mass of Bled Steam

Neglecting condensate pump work and pump works between heaters.

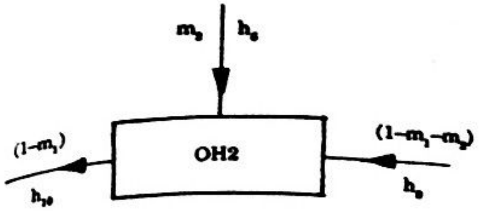
$h_{B8} \approx h_8 \qquad h_{B9} \approx h_9 \qquad h_{B10} \approx h_{10}$

Last heater:


m1
h2
OH3
1Kg
h11
(1–m1)
h10

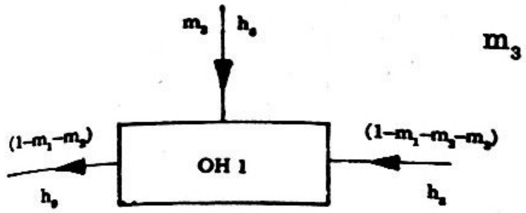
$$m_1 = (1)\left[\frac{h_{11} - h_{10}}{h_2 - h_{10}}\right]$$

Second heater:


m2
h5
OH2
(1–m1)
h10
(1–m1–m2)
h9

$$m_2 = (1-m_1)\left[\frac{h_{10} - h_9}{h_5 - h_9}\right]$$

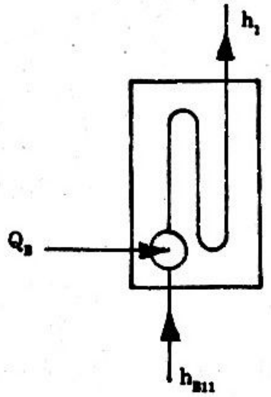
First heater:

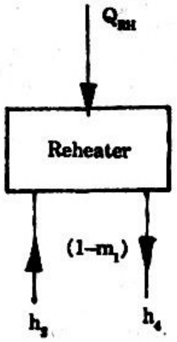

m3
h6
OH 1
(1–m1–m2)
h9
(1–m1–m2–m3)
h8

$$m_3 = (1 - m_1 - m_2)\left[\frac{h_9 - h_8}{h_6 - h_8}\right]$$

### b. Work

Cycle Work, $W_{net}$



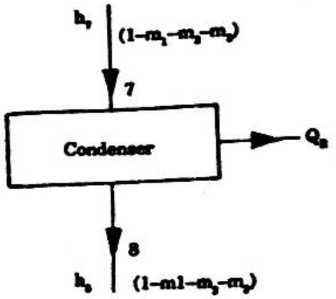

QRH
Reheater
(1–m1)
h3
h4

$$Q_B = h_1 - h_{B11} = h_1 - h_{11} - W_{p4} \qquad Q_{Rh} = (1 - m_1)(h_4 - h_3)$$

$$= h_1 - h_{11} - v_{f11}(p_{B11} - p_{11})$$

$$Q_A = Q_{Boiler} + Q_{Reheater}$$

$$= h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (1 - m_1)(h_4 - h_3)$$


h7
(1-m1-m2-m3)
7
Condenser
QR
8
h8
(1-m1-m2-m3)

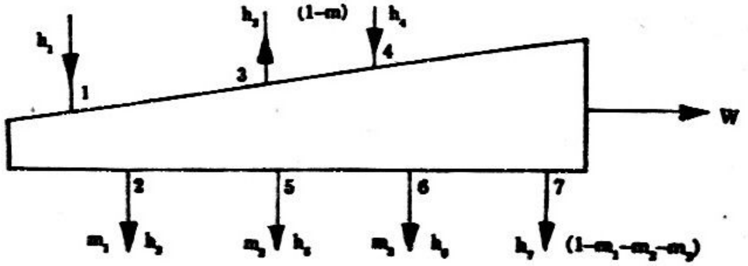
$$Q_R = (1 - m_1 - m_2 - m_3)\ (h_7 - h_8)$$

Therefore, the net cycle work based from $Q_A$ and $Q_R$ is

$$W_{net} = Q_A - Q_R$$

$$= h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (1 - m_1)(h_4 - h_3) - (1 - m_1 - m_2 - m_3)(h_7 - h_8)$$

Turbine Work, W


h1
1
h3
3
(1-m)
h4
4
W
2
5
6
7
m1 h2
m2 h5
m3 h6
h7 (1-m1-m2-m3)

First Method:

$$E_{in} = E_{out}$$

$$h_1 + (1-m_1)\,h_4 = m_1h_2 + (1-m_1)\,h_3 + m_2h_5 + m_3h_6 + (1 - m_1 - m_2 - m_3)\,h_7 + W$$

$$W = h_1 + (1-m_1)\,h_4 - m_1h_2 - (1-m_1)h_3 - m_2h_5 - m_3h_6 - (1-m_1-m_2-m_3)h_7$$

Second Method:

$$W = W_{1-2} + W_{2-3} + W_{4-5} + W_{5-6} + W_{6-7}$$

$$W = (h_1 - h_2) + (1-m_1)(h_2 - h_3) + (1-m_1)(h_4 - h_5) + (1-m_1-m_2)(h_5 - h_6) + (1-m_1-m_2-m_3)(h_6 - h_7)$$

c. Thermal Efficiency, $e_c$

$$e_c = \frac{W_{net}}{Q_A}$$

$$= \frac{h_1-h_{11}-v_{f11}(p_{B11}-p_{11}) + (1-m_1)(h_4-h_3) - (1-m_1-m_2-m_3)(h_7-h_8)}{h_1-h_{11}-v_{f11}(p_{B11} - p_{11}) + (1-m_1)(h_4-h_3)}$$

CASE 3.

The same as Case 2 except that the three extraction points occur after the reheating.

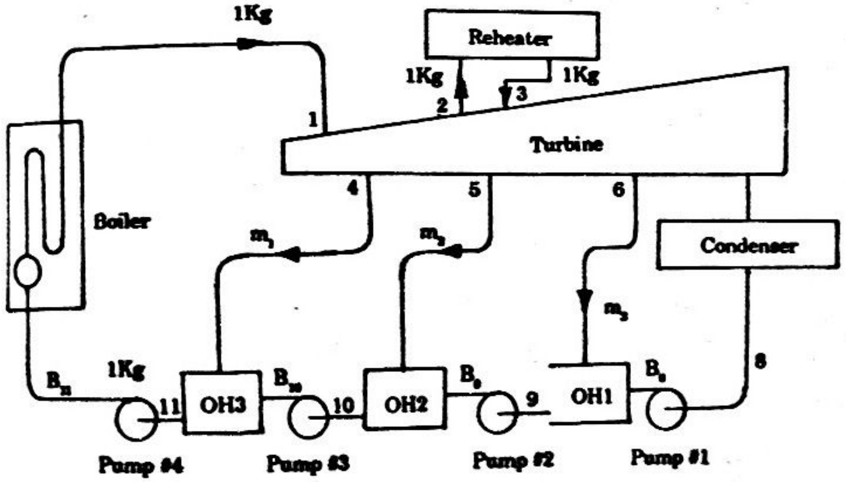

1Kg
Reheater
1Kg
1Kg
2
3
1
Turbine
4
5
6
Boiler
Condenser
m1
m2
m3
8
B11
1Kg
11
OH3
B10
10
OH2
B9
9
OH1
B8
Pump #4
Pump #3
Pump #2
Pump #1

Fig. 3-18. Plant Layout of Case 3.

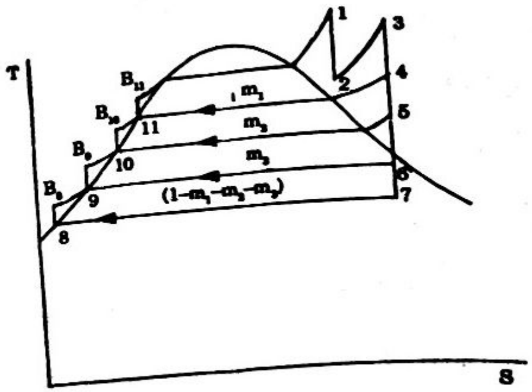
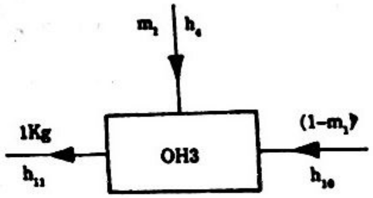

T
s
1
2
3
4
5
6
7
8
9
10
11
B11
B10
B9
B8
m1
m2
m3
(1-m1-m2-m3)

Fig. 3-19. Ts diagram of Case 3.

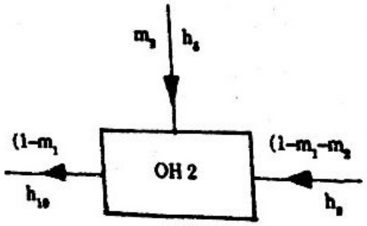
a. **Mass of Bled Steam**

**Last heater:**


m1
h4
OH3
1Kg
h11
(1-m1)
h10

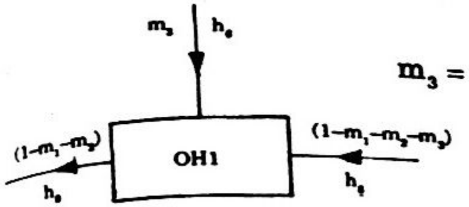
$$m_1 = \frac{h_{11} - h_{10}}{h_4 - h_{10}}$$

**Second heater:**


m2
h5
OH 2
(1-m1)
h10
(1-m1-m2)
h9

$$m_2 = (1 - m_1)\left[\frac{h_{10} - h_9}{h_5 - h_9}\right]$$

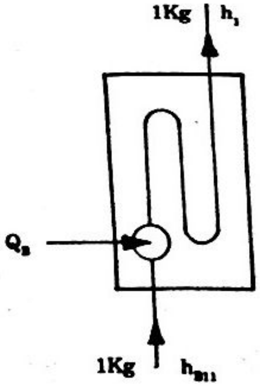
**Third heater:**

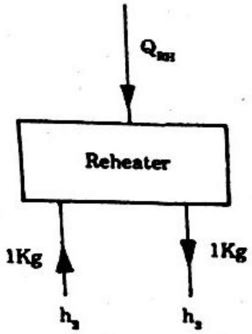

m3
h6
OH1
(1-m1-m2)
h9
(1-m1-m2-m3)
h8

$$m_3 = (1 - m_1 - m_2 - m_3)\left[\frac{h_9 - h_8}{h_6 - h_8}\right]$$

b. **Work**

**Cycle Work, $W_{net}$**


1Kg
h1
QB
1Kg
hB11


QRH
Reheater
1Kg
h2
1Kg
h3

$$Q_B = h_1 - h_{B11} = h_1 - h_{11} - W_{p4} \qquad Q_{RH} = h_3 - h_2$$

$$= h_1 - h_{11} - v_{f11}(p_{B11} - p_{11})$$

$$Q_A = Q_{Boiler} + Q_{Reheater}$$

$$= h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (h_3 - h_2)$$

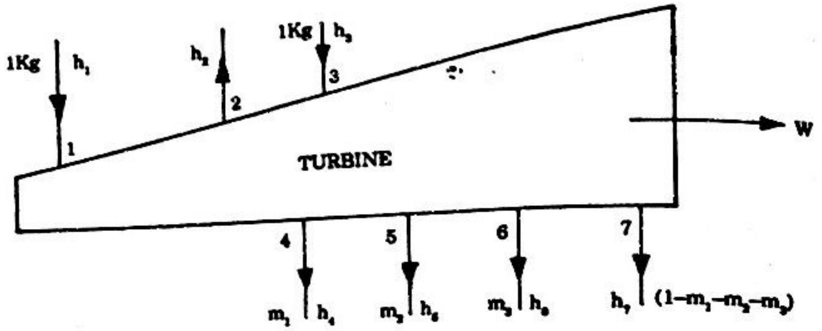
$$Q_R = (1 - m_1 - m_2 - m_3)(h_7 - h_8)$$

(The same as in Case 2.)

$$W_{net} = Q_A - Q_R$$

$$= [h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (h_3 - h_2)] - (1 - m_1 - m_2 - m_3)(h_7 - h_8)$$

Turbine Work, W


1Kg
$h_1$
$h_2$
1Kg
$h_3$
1
2
3
TURBINE
W
4
5
6
7
$m_1$ $h_4$
$m_2$ $h_5$
$m_3$ $h_6$
$h_7$ $(1-m_1-m_2-m_3)$

$$E_{in} = E_{out}$$

$$h_1 + h_3 = h_2 + m_1h_4 + m_2h_5 + m_3h_6 + m_3h_6 + (1 - m_1 - m_2 - m_3)\,h_7 + W$$

$$W = h_1 + h_3 - [h_2 + m_1h_4 + m_2h_5 + m_3h_6 + (1 - m_1 - - m_2 - m_3)\,h_7]$$

Another method

$$W = W_{1-2} + W_{2-3} + W_{3-4} + W_{5-6} + W_{6-7}$$

$$W = (h_1 - h_2) + (h_3 - h_4) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6) + (1 - m_1 - m_2 - m_3)\ h_6 - h_7)$$

Thermal Efficiency, $e_c$

$$e_c = \frac{W_{net}}{Q_A}$$

$$= \frac{[h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (h_3 - h_2)] - (1-m_1-m_2-m_3)(h_7 - h_8)}{h_1 - h_{11} - v_{f11}(p_{B11} - p_{11}) + (h_3 - h_2)}$$

## CASE 4.

Assume an ideal reheat regenerative cycle: after some expansion, part of the steam is extracted for feedwater heating; the remainder are withdrawn and reheated to the original temperature; after further expansion, a second extraction occurs; then expansion to exhaust. Write the equations for (a) the quantity of steam extracted and (b) the turbine work.

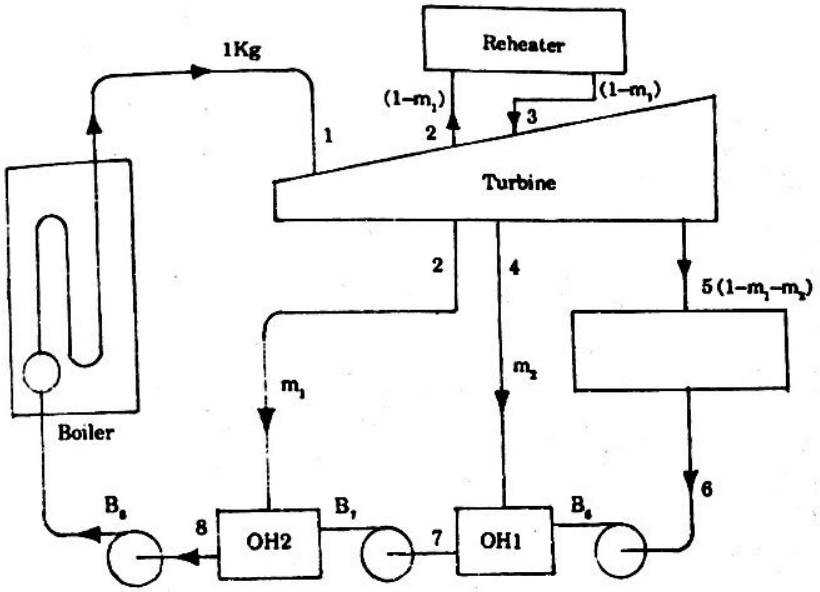

1Kg
Reheater
$(1-m_1)$
$(1-m_1)$
1
2
3
Turbine
2
4
$5\ (1-m_1-m_2)$
$m_1$
$m_2$
Boiler
6
$B_8$
$B_7$
$B_6$
8
OH2
7
OH1

Fig. 3-20. Plant Layout of Case 4

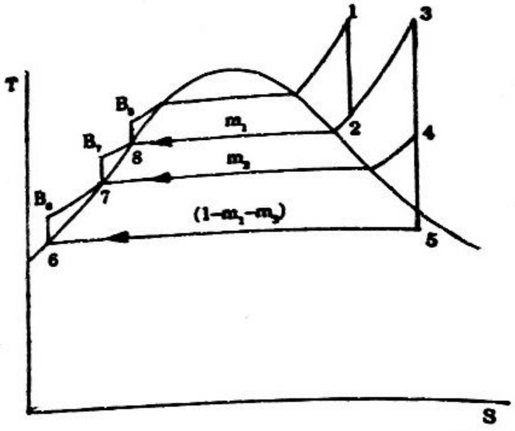
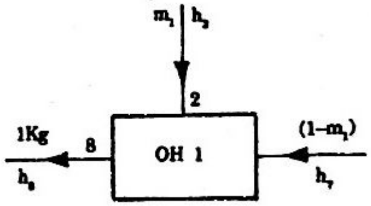

T
S
1
2
3
4
5
6
7
8
B₆
B₇
B₈
m₁
m₂
(1−m₁−m₂)

Fig. 3-21. Ts diagram of Case 4.

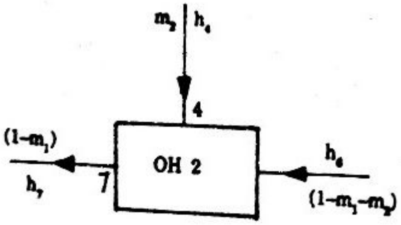
a. Mass of Bled Steam

Open heater No. 1:


m₁
h₂
2
1Kg
8
h₈
OH 1
(1−m₁)
h₇

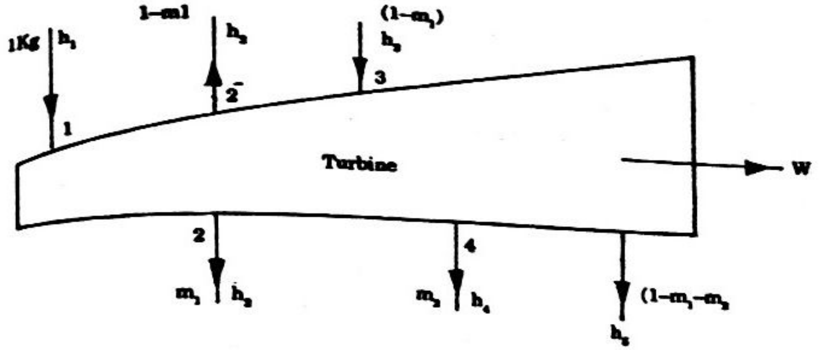
$$m_1 = \frac{h_8 - h_7}{h_2 - h_7}$$

Open heater No. 2:


m₂
h₄
4
(1−m₁)
h₇
7
OH 2
h₆
(1−m₁−m₂)

$$m_2 = (1 - m_1)\,[\frac{h_7 - h_6}{h_4 - h_6}]$$

b. Turbine Work, W


1Kg
h₁
1
1−m1
h₂
2
(1−m₁)
h₃
3
Turbine
W
2
m₁
h₂
4
m₂
h₄
(1−m₁−m₂
h₅

$$E_{in} = E_{out}$$

$$h_1 + (1-m_1)h_3 = m_1h_2 + (1-m_1)h_2 + m_2h_4 + (1-m_1-m_2)h_5 + W$$

$$W = (h_1 - h_2) + (1-m_1)h_3 - m_2h_4 - (1-m_1-m_2)h_5$$

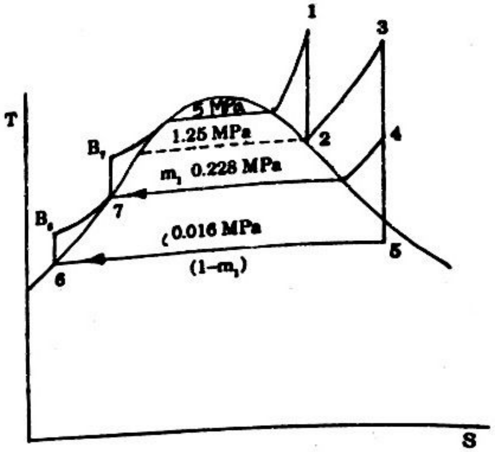
or

$$W = W_{1-2} + W_{3-4} + W_{4-5}$$

$$= (h_1 - h_2) + (1-m_1)(h_3 - h_4) + (1-m_1-m_2)(h_4 - h_5)$$

(3-14) Steam at 5 MPa and 365°C enters a turbine and expands until it becomes saturated. The steam is withdrawn and reheated to 330°C. After expansion in the turbine to 150°C, $m_1$ kg is extracted for feedwater heating. The remaining steam expands to the condenser pressure of 0.016 MPa. For 1 kg of steam, find $W_{net}$, $e_c$, $e_e$, and the ideal steam rate.

**SOLUTION:**



$h_1$ = h at 5 MPa and 365°C = 3108 kJ/kg
$h_2$ = h at $s_2$ equal to $s_1$ and saturated = 2786 kJ/kg
$h_3$ = h at 1.25 MPa and 330°C = 3110 kJ/kg
$h_4$ = h at 150°C (0.28 MPa) and $s_4$ equal to $s_3$ = 2762 kJ/kg
$h_5$ = h at 0.016 MPa and $s_5$ equal to $s_4$ = 2315 kJ/kg
$h_6$ = $h_f$ at 0.016 MPa = 231.56 kJ/kg
$h_7$ = $h_f$ at 0.28 MPa = 551.48 kJ/kg
$v_{f6}$ = $v_f$ at 0.016 MPa = 0.0010147 m³/kg
$v_{f7}$ = $v_f$ at 0.28 MPa = 0.0010709 m³/kg
$W_{p2} = v_{f7}(p_1 - p_7) = 0.0010709\ (5000 - 280) = 5.05$ kJ/kg
$h_{B7} = h_7 + W_{p2} = 551.48 + 5.05 = 556.5$ kJ/kg
$\Sigma W_p = v_{f6}(p_1 - p_6) = 0.0010147\ (5000 - 16) = 5.06$ kJ/kg

Heater

$$m_1 = \frac{h_7 - h_6}{h_4 - h_6} = \frac{551.48 - 231.56}{2762 - 231.56} = 0.1264 \text{ kg/kg}$$

$$W = (h_1 - h_2) + (h_3 - h_4) + (1 - m_1)(h_4 - h_5)$$

$$= (3108 - 2786) + (3110 - 2762) + (1 - 0.1264)(2762 - 2315)$$

$$= 1060.5 \text{ kJ/kg}$$

$$W_{net} = W - \Sigma W_p = 1060.5 - 5.06 = 1055.4 \text{ kJ/kg}$$

$$Q_A = h_1 - h_{B7} + h_3 - h_2$$

$$= 3108 - 551.48 + 3110 - 2786$$

$$= 2875.5 \text{ kJ/kg}$$

$$E_c = h_1 - h_{f7} + h_3 - h_2$$

$$= 3108 - 551.48 + 3100 - 2786$$

$$= 2880.5 \text{ kJ/kg}$$

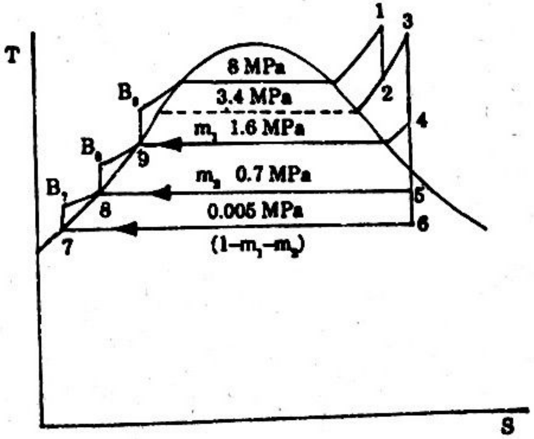
$$e_c = \frac{W_{net}}{Q_A} = \frac{1055.4}{2875.5} = 0.3670 \text{ or } 36.70\%$$

$$e_e = \frac{W}{E_c} = \frac{1060.5}{2880.5} = 0.3682 \text{ or } 36.82\%$$

$$m = \frac{3600}{W} = \frac{3600}{1060.5} = 3.39 \text{ kg/kwh}$$

(3-15) In an ideal reheat-regenerative cycle, steam enters the engine at 8 MPa and 400°C. After expansion to 280°C the steam is withdrawn and reheated to 340°C. Extractions for feedwater heating occur at 1.6 MPa and 0.70 MPa and the condenser pressure is 0.005 MPa. For the ideal cycle find (a) $W_{net}$ and $e_c$. For the ideal engine, find (b) W and $e_e$.

**SOLUTION:**



$h_1$ = h at 8 MPa and 400°C = 3138 kJ/kg

$h_2$ = h at 280°C (3.4 MPa) and $s_2$ equal to $s_1$ = 2926 kJ/kg

$h_3$ = h at 3.4 MPa and 340°C = 3082 kJ/kg

$h_4$ = h at 1.6 MPa and $s_4$ equal to $s_3$ = 2897 kJ/kg

$h_5$ = h at 0.7 MPa and $s_5$ equal to $s_4$ = 2732 kJ/kg

$h_6$ = h at 0.005 MPa and $s_6$ equal to $s_5$ = 2024 kJ/kg

$h_7$ = $h_f$ at 0.005 MPa = 137.82 kJ/kg

$h_8$ = $h_f$ at 0.7MPa = 697.22 kJ/kg

$h_9$ = $h_f$ at 1.6 MPa = 858.79 kJ/kg

$v_{f7}$ = $v_f$ at 0.005 MPa = 0.0010053 m³/kg

$v_{f9}$ = $v_f$ at 1.6 MPa = 0.0011587 m³/kg

$W_{p3} = v_{f9}(p_{B9} - p_9) = 0.0011587\,(8000 - 1600) = 7.4$ kJ/kg

$\Sigma W_p = v_{f7}(p_{B9} - p_7) = 0.0010053\,(8000 - 5) = 8.04$ kJ/kg

$h_{B9} = h_9 + W_{p3} = 858.79 + 7.4 = 866.19$ kJ.kg

$$(1 - m_1)(h_9 - h_8) = m_1(h_4 - h_8)$$

$$m_1 = \frac{h_9 - h_8}{h_4 - h_8} = \frac{858.79 - 697.22}{2897 - 697.22}$$

$$= 0.07345 \text{ kg/kg}$$

$$(1 - m_1 - m_2)(h_8 - h_7) = m_2(h_5 - h_8)$$

$$m_2 = \frac{(1 - m_1)(h_8 - h_7)}{h_5 - h_7}$$

$$= \frac{(1 - 0.07345)(697.22 - 137.82)}{2732 - 137.82}$$

$$= 0.1998 \text{ kg/kg}$$

(a) $W = h_1 - h_2 + h_3 - h_4 + (1 - m_1)(h_4 - h_5)$

$+ (1 - m_1 - m_2)(h_5 - h_6)$

$= 3138 - 2926 + 3082 - 2897 + (1 - 0.07345)$

$(2897 - 2732) + (1 - 0.07345 - 0.1998)(2732 - 2024)$

$= 1064.4$ kJ/kg

$W_{net} = W - \Sigma W_p = 1064.4 - 8.04 = 1056.4$ kJ/kg

$Q_A = h_1 - h_{B9} + h_3 - h_2$

$= 3138 - 866.19 + 3082 - 2926 = 2427.8$ kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{1056.4}{2427.8} = 0.4351 \text{ or } 43.51\%$$

(b) $W = 1064.4$ kJ/kg

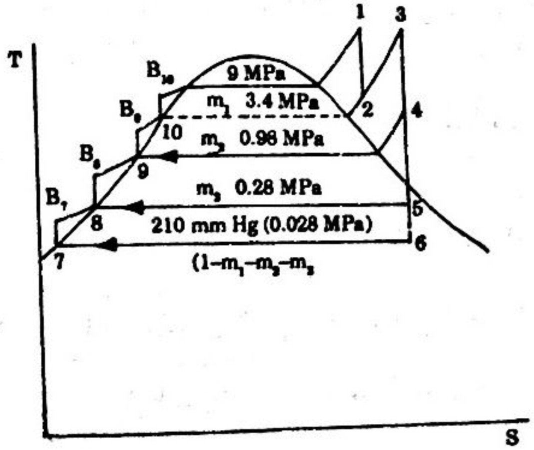
$E_c = h_1 + h_3 - h_2 - h_{f9}$

$= 3138 + 3082 - 2926 - 858.79 = 2435.2$ kJ/kg

$$e_e = \frac{W}{E_c} = \frac{1064.4}{2435.2} = 0.4371 \text{ or } 43.71\%$$

(3-16) An ideal reheat regenerative turbo-generator unit receives 9.0 MPa of throttle steam at 390°C. Reheat occurs at the first bleeding point of 3.40 MPa. The second and third extractions are at 0.98 MPa and 0.28 MPa, respectively. For a condenser vacuum of 550 mm Hg, find (a) the cycle thermal efficiency and (b) the engine thermal efficiency.

**SOLUTION:**



$h_1$ = h at 9.0 MPa and 390°C = 3088 kJ/kg
$h_2$ = h at 3.40 MPa and $s_2$ equal to $s_1$ = 2858 kJ/kg
$h_3$ = h at 3.40 MPa and 390°C = 3201 kJ/kg
$h_4$ = h at 0.98 MPa and $s_4$ equal to $s_3$ = 2885 kJ/kg
$h_5$ = h at 0.28 MPa and $s_5$ equal to $s_4$ = 2644 kJ/kg
$h_6$ = h at 0.028 MPa and $s_6$ equal to $s_5$ = 2292 kJ/kg
$h_7$ = $h_f$ at 0.028 MPa = 282.62 kJ/kg
$h_8$ = $h_f$ at 0.28 MPa = 551.48 kJ/kg
$h_9$ = $h_f$ at 0.98 MPa = 758.94 kJ/kg
$h_{10}$ = $h_f$ at 3.40 MPa = 1041.82 kJ/kg
$v_{f10}$ = $h_f$ at 3.40 MPa = 0.0012311 m³/kg
$v_{f7}$ = $v_f$ at 0.028 MPa = 0.0016214 m³/Kg
$W_{p4} = v_{f10}(p_1 - p_{10}) = 0.0012311\ (9000 - 3400) = 6.89$ kJ/kg
$\Sigma W_p = v_{f7}(p_1 - p_7) = 0.0016214\ (9000 - 28) = 9.16$ kJ/kg
$h_{B10} = h_{10} + W_{p4} = 1041.82 + 6.89 = 1048.71$ kJ/kg

Heater No. 3

$$m_1 = \frac{h_{10} - h_9}{h_2 - h_9} = \frac{1041.82 - 758.94}{2858 - 758.94} = 0.1348 \text{ kg/kg}$$

Heater No. 2

$$m_2 = \frac{(1 - m_1)(h_9 - h_8)}{h_4 - h_8} = \frac{(1-0.1348)\ (758.94-551.48)}{2885 - 551.48}$$

$$= 0.07692 \text{ kg/kg}$$

Heater No. 1

$$m_3 = \frac{(1 - m_1 - m_2)(h_8 - h_7)}{h_5 - h_7}$$

$$= \frac{(1-0.1348 - 0.07692)\ (551.48 - 282.62)}{2644 - 282.62}$$

$$= 0.08975 \text{ kg/kg}$$

(a) $W = (h_1 - h_2) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

$+ (1 - m_1 - m_2 - m_3)(h_5 - h_6)$

$= (3088 - 2858) + (1 - 0.1348)\ (3201 - 2885)$

$+ (1 - 0.1348 - 0.07692)\ (2885 - 2644)$

$+ (1 - 0.1348 - 0.07692 - 0.08975)\ (2644 - 2292)$

$= 939.3$ kJ/kg

$W_{net} = W - \Sigma W_p = 939.3 - 9.16 = 930.1$ kJ/kg

$Q_A = h_1 - h_{B10} + (1 - m_1)(h_3 - h_2)$

$= 3088 - 1048.71 + (1 - 0.1348)\ (3201 - 2858)$

$= 2336.1$ kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{930.1}{2336.1} = 0.3981 \text{ or } 39.81\%$$

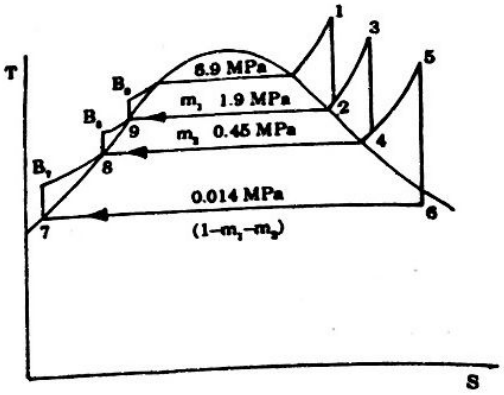
(b) $E_c = h_1 - h_{f8} + (1 - m_1)(h_3 - h_2)$

$= 3088 - 282.62 + (1 - 0.1348)\ (3201 - 2858)$
$= 3102.1$ kJ/kg

$$e_e = \frac{W}{E_c} = \frac{939.3}{3102.1} = 0.3028 \text{ or } 30.28\%$$

(3-17) Steam at 6.9 MPa and 470°C enters a turbine and expands to 1.9 MPa where $m_1$ is extracted for feedwater heating, the remainder is withdrawn and reheated to 360°C. Expansion then occurs to 0.45 MPa, where $m_2$ is extracted for feedwater heating. The remaining steam is withdrawn and reheated to 290°C. Final expansion now occurs to a condenser pressure of 0.014 MPa. For an ideal cycle and 1 kg of throttle steam, find (a) $W_{net}$ and $e_c$. For the ideal engine find (b) W and $e_e$. (c) For an engine output of 33,500 kW, what is the steam flow in kg/h?

**SOLUTION:**



$h_1$ = h at 6.9 MPa and 470°C = 3338.2 kJ/kg
$h_2$ = h at 1.9 MPa and $s_2$ equal to $s_1$ = 2978 kJ/kg
$h_3$ = h at 1.9 MPa and 360°C = 3161 kJ/kg
$h_4$ = h at 0.45 MPa and $s_4$ equal to $s_3$ = 2816 kJ/kg
$h_5$ = h at 0.45 MPa and 290°C = 3045 kJ/kg
$h_6$ = h at 0.014 MPa and $s_6$ equal to $s_5$ = 2415 kJ/kg
$h_7$ = hf at 0.014 MPa = 219.99 kJ/kg
$h_8$ = $h_f$ at 0.45 MPa = 623.25 kJ/kg
$h_9$ = $h_f$ at 1.9 MPa = 897.02 kJ/kg
$v_{f7}$ = $v_f$ at 0.014 MPa = 0.0010134 m³/kg
$v_{f9}$ = $v_f$ at 1.9 MPa = 0.0011724 m³/kg
$W_{p3} = v_{f9}(p_{B9} - p_9) = 0.0011724\,(6900 - 1900) = 5.86$ kJ/kg
$\Sigma W_p = v_{f7}\,(p_{B9} - p_7) = 0.0010134\,(6900 - 14) = 6.98$ kJ/kg
$h_{B9} = h_9 + W_{p3} = 897.02 + 5.86 = 902.88$ kJ/kg

$$(1 - m_1)(h_9 - h_8) = m_1(h_2 - h_9)$$

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8} = \frac{897.02 - 623.25}{2978 - 623.25}$$

$$= 0.1163 \text{ kg/kg}$$

$$(1 - m_1 - m_2)(h_8 - h_7) = m_2(h_4 - h_8)$$

$$m_2 = \frac{(1 - m_1)(h_8 - h_7)}{h_4 - h_7}$$

$$= \frac{(1 - 0.1163)(623.25 - 219.99)}{2816 - 219.99}$$

$$= 0.1373 \text{ kg/kg}$$

$$W = (h_1 - h_2) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_5 - h_6)$$

$$= (3338.2 - 2978) + (1 - 0.1163)(3161 - 2816)$$

$$+ (1 - 0.1163 - 0.1373)(3045 - 2415)$$

$$= 1135.3 \text{ kJ/kg}$$

$$E_c = (h_1 - h_{f9}) + (1 - m_1)(h_3 - h_2) + (1 - m_1 - m_2)(h_5 - h_4)$$

$$= (3338.2 - 897.02) + (1 - 0.1163)(3161 - 2978)$$

$$+ (1 - 0.1163 - 0.1373)(3045 - 2816)$$

$$= 2773.8 \text{ kJ/kg}$$

$$Q_A = (h_1 - h_{B9}) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_5 - h_4)$$

$$= (3338.2 - 902.88) + (1 - 0.1163)(3161 - 2978)$$

$$+ (1 - 0.1163 - 0.1373)(3045 - 2816) = 2767.9 \text{ kJ/kg}$$

$$W_{net} = W - \Sigma W_p = 1135.3 - 6.98 = 1128.3 \text{ kJ/kg}$$

(a) $W_{net}$ = 1128.3 kJ/kg

$$e_c = \frac{W_{net}}{Q_A} = \frac{1128.3}{2767.9} = 0.4076 \text{ or } 40.76\%$$

(b) W = 1135.3 kJ/kg

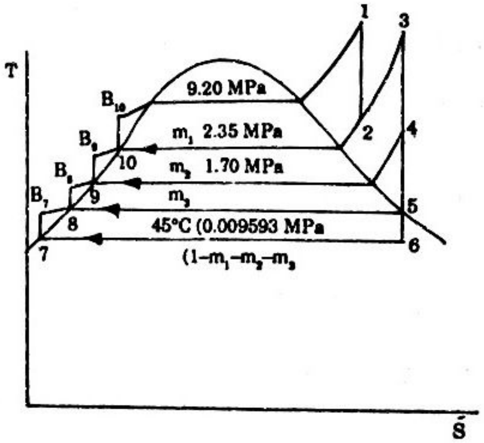
$$e_e = \frac{W}{E_c} = \frac{1135.3}{2773.8} = 0.4093 \text{ or } 40.93$$

$$(c)\ m = \frac{3600}{W} = \frac{3600}{1135.3} = 3.17 \text{ kg/kwh}$$

$$m = (33{,}500 \text{ kw})(3.17 \text{ kg/kwh}) = 106{,}195 \text{ kg/h}$$

(3-18) A 125 MW reheat-regenerative turbo-generator unit receives 500000 kg/hr of steam at 9.20 MPa and 440°C. Generator efficiency is 95%. Condensate leaves at 45°C. For ideal cycle, first extraction is at 2.35 MPa; second extraction is at 1.70 MPa; and the third extraction is dry and saturated. For actual cycle with the same extraction pressure except that pressure drops through reheater tubes by 0.25 MPa, entering at temperature of 260°C. Actual feedwater temperature is 215°C. Calculate (a) the ideal engine thermal efficiency, (b) the cycle thermal efficiency without pressure drop thru reheater, (c) the actual mass of bled steam for last heater per kg. of throttled steam and the corresponding Brake thermal efficiency, (d) the ideal work of engine with pressure drop thru reheater tubes and the thermal efficiency, and (e) the combined engine efficiency for cycle without pressure drop thru reheater and with pressure drop.

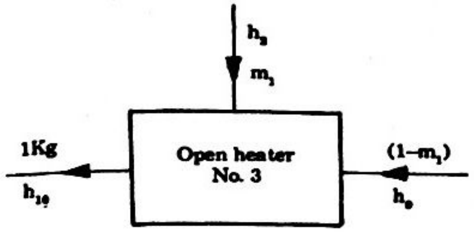
**SOLUTION:**



$h_1$ = h at 9.20 MPa and 440°C = 3226.6 kJ/kg
$h_2$ = h at 2.35 MPa and $s_2$ equal to $s_1$ = 2880 kJ/kg
$h_3$ = h at 2.35 MPa and 440°C = 3331 kJ/kg
$h_4$ = h at 1.70 MPa and $s_4$ equal to $s_3$ = 3230 kJ/kg

$h_5$ = h at $s_5$ equal to $s_3$ and saturated = 2700 kJ/kg
$h_6$ = h at 0.009593 MPa and $s_6$ equal to $s_3$ = 2270 kJ/kg
$h_7$ = $h_f$ at 45°C = 188.45 kJ/kg
$h_8$ = $h_f$ at 0.174 MPa = 485.9 kJ/kg
$h_9$ = $h_f$ at 1.70 MPa = 872.06 kJ/kg
$h_{10}$ = $h_f$ at 2.35 MPa = 946.97 kJ/kg
$v_{f10}$ = $v_f$ at 45°C = 0.0010099 m³/kg
$v_{f7}$ = $v_f$ at 2.35 MPa = 0.0011913 m³/kg
$W_{p1}$ = $v_{f10}(p_{B10} - p_{10}) = 0.0011913\ (9200 - 2350) = 8.16$ kJ/kg
$\Sigma W_p$ = $v_{f7}(p_{B10} - p_7) = 0.0010099\ (9200 - 9.593) = 9.2814$ kJ/kg

For open heater no. 3



$$m_1 = \frac{(h_{10} - h_9)}{(h_2 - h_9)} = \frac{946.99 - 872.06}{2880 - 872.06} = 0.0373 \text{ kg/kg of}$$

throttle steam

For open heater no. 2

$$m_2 = (1 - m_1)\left[\frac{h_9 - h_8}{h_4 - h_8}\right] = (1 - 0.0373)\left[\frac{872.06 - 485.9}{3230 - 485.9}\right]$$

$$= 0.1335 \text{ kg/kg of throttle steam}$$

For open heater no. 1

$$m_3 = (1 - m_1 - m_2)\left[\frac{h_8 - h_7}{h_6 - h_7}\right] = (1 - 0.0373 - 0.1335)$$

$$\left[\frac{485.9 - 188.45}{3230 - 188.45}\right]$$

$= 0.098$ kg/kg of throttle steam

$$W = h_1 - h_2 + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5) + (1 - m_1 - m_2 - m_3)(h_5 - h_6)$$

$$= 3226.6 - 2880 + (1 - 0.0373)(3331 - 3230) + (1 - 0.00373 - 0.1355)(3230 - 2700) + (1 - 0.0373 - 0.1355 - 0.098)(2700 - 2270)$$

$$= 1196.5 \text{ kJ/kg}$$

$$W_{net} = W - \Sigma W_p = 1196.5 - 9.2814 = 1187.2 \text{ kJ/kg}$$

$$E_c = h_1 - h_{10} + (1 - m_1)(h_3 - h_2)$$

$$= 3226.6 - 946.97 + (1 - 0.0373)(3331 - 2880)$$

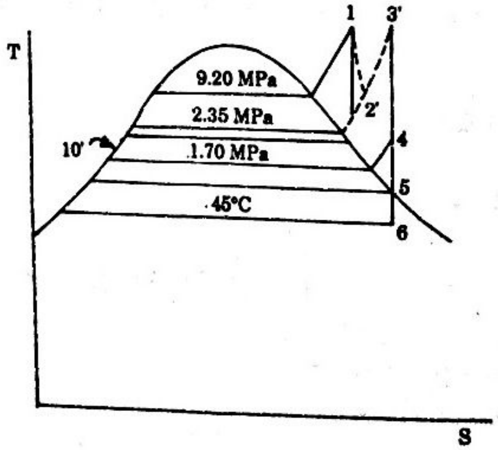
$$= 2713.83 \text{ kJ/kg}$$

$$Q_A = E_c - Wp_4 = 2713.83 - 816 = 2705.67 \text{ kJ/kg}$$

(a) $e_e = \dfrac{W}{E_c} = \dfrac{1196.5}{2713.83} = 0.441$ or 44.1%

(b) $e_c = \dfrac{W_{net}}{Q_A} = \dfrac{1187.72}{2705.67} = 0.439$ or 43.9%

(c)

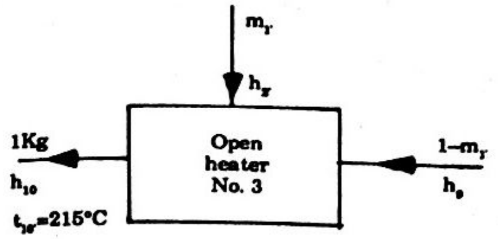


*For actual cycle*

$$m_k = \frac{500{,}000}{125{,}000} = 4 \text{ kg/kwh}$$

$$W_k = \frac{3600}{m_k} = \frac{3600}{4} = 900 \text{ kJ/kwh}$$

$h_{10'} =$ $h_f$ at 215°C = 920.72 kJ/kg
$h_{2'} =$ h At 2.35 MPa and 260°C = 2913.6 kJ/kg
$h_{3'} =$ h at 2.10 MPa and 440°C = 3334.2 kJ/kg



$m_{1'} = \dfrac{h_{10'} - h_9}{h_{2'} - h_9} = \dfrac{920.62 - 872.06}{2913.6 - 872.06} = 0.016$ kg/kg of throttle steam

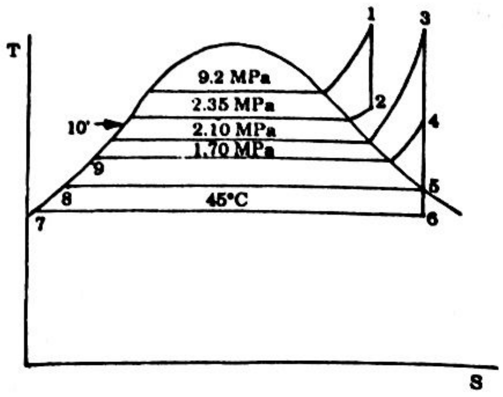
$$E_c = h_1 - h_{10'} + (1 - m_{1'})(h_{3'} - h_{2'})$$

$$= 3226.6 - 920.62 + (1 - 0.016)(3334.2 - 2913.6)$$
$$= 2719.85 \text{ kJ/kg}$$

$$W_B = \frac{W_k}{n_g} = \frac{900}{0.95} = 947.4 \text{ kJ/kg}$$

$e_B = \dfrac{W_B}{E_{c'}} = \dfrac{947.4}{2719.85} = 0.348$ or 34.8%

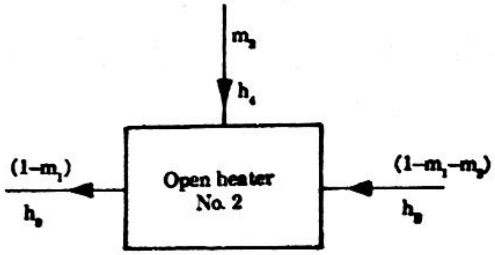
(d) For ideal cycle with pressure drop thru reheater tubes:



$h_1$ = h at 9.20 MPa and 440°C = 3226.6 kJ/kg
$h_2$ = h at 2.35 MPa and $s_2$ equal to $s_1$ = 2880 kJ/kg

with pressure drop of 0.25 MPa thru reheater tubes

$h_3$ = h at 2.10 MPa and 440°C = 3334.2 kJ/kg
$h_4$ = h at 1.70 MPa and $s_4$ equal to $s_3$ 3268 kJ/kg
$h_5$ = h att $s_5$ equal to $s_3$ and saturated = 2693 kJ/kg
$h_6$ = h at 0.009593 MPa and $s_6$ equal to $s_3$ = 2286 kJ/kg
$h_7$ = $h_f$ at 45°C = 188.45 kJ/kg
$h_8$ = $h_f$ with $h_5$ equal to $h_g$ = 464.6 kJ/kg
$h_9$ = $h_f$ at 1.70 MPa = 872.06 kJ/kg



$$m_2 = (1 - m_1)\left[\frac{h_9 - h_8}{h_4 - h_8}\right] = (1 - 0.0373)\left[\frac{872.06 - 464.6}{3268 - 464.6}\right]$$

$$= 0.14 \text{ kg/kg of throttle}$$

Heat balance for heater no. 1

$$m_3 = (1 - m_1 - m_2)\left[\frac{h_8 - h_7}{h_5 - h_7}\right] = (1 - 0.0373 - 0.14)$$

$$\left[\frac{464.6 - 188.45}{2693 - 188.45}\right] = 0.0907 \text{ kg/kg of throttle steam}$$

$$W = h_1 - h_2 + (1 - m_1)(h_3 - h4) + (1 - m_1 - m_2)(h_4 - h_5) + (1 - m_1 - m_2 - m_3)(h_5 - h_6)$$

$$= 3226.6 - 2880 + (1 - 0.0373)(3334.2 - 3268) + (1 - 0.0373 - 0.14)(3268 - 2693) + (1 - 0.0373 - 0.14 - 0.0907)(2693 - 2286)$$

$$= 1181.33 \text{ kJ/kg}$$

$$E_c = h_1 - h_{10} + (1 - m_1)(h_3 - h_2)$$
$$= 3226.6 - 946.97 + (1 - 0.0373)(3334.2 - 2880)$$
$$= 2716.9 \text{ kJ/kg}$$

$$m = \frac{3600}{W} = \frac{3600}{1181.33} = 3.047 \text{ kg/kwh}$$

$$HR = (m)(E_c) = (3.047)(2716) = 8278.4 \text{ kJ/kg}$$

$$e_e = \frac{3600}{HR} = \frac{3600}{8278.4} = 0.435 \text{ or } 43.5\%$$

(e) $n_k = \frac{W_k}{W} = \frac{900}{1196.5} = 0.752$ or 75.2%
(without pressure drop)

$n_k = \frac{W_k}{W} = \frac{900}{1181.33} = 0.762$ or 76.2%
(with pressure drop)

## The Incomplete-Expansion Cycle

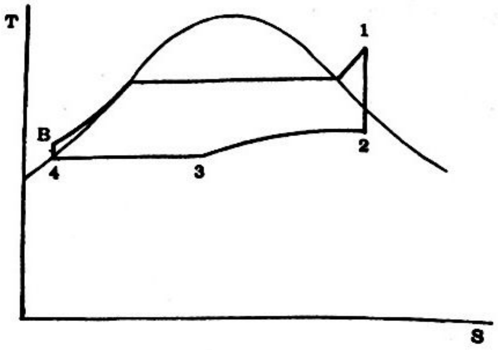


Fig. 3-22. Incomplete-Expansion Cycle.

The ideal incomplete-expansion cycle is composed of the following processes.

1-2: isentropic expansion process in the engine, $s = C$
2-3: constant volume rejection of heat process in the engine, $V = C$
3-4: constant pressure rejection of heat process in the condenser, $p = C$
4-B: reversible adiabatic pumping process, $s = C$
B-1: constant pressure addition of heat process in the boiler, $p = C$

Engine Work, W

Recalling isentropic and isometric processes.

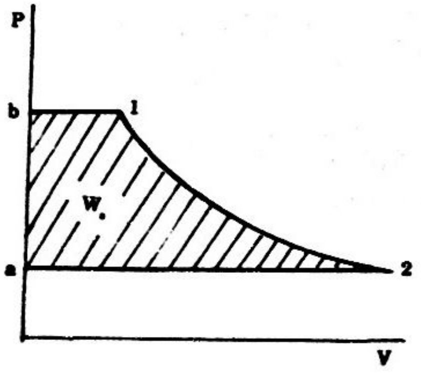


Fig. 3-23. Isentropic Process

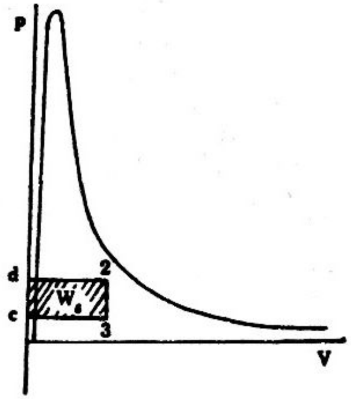


Fig. 3-24. Isometric Process

Neglecting ΔP and ΔK, the area behind the curve on the pv plane represents the work of a steady flow process.

For isentropic process:

$W_s =$ area behind the curve, area 1–2–a–b–1

$W_s = h_1 - h_2$

For isometric process:

$W_s =$ area behind the curve, area 2-3-c-d-2

$W_s = v_2 (p_2 - p_3)$

For incomplete-expansion engine with zero clearance:

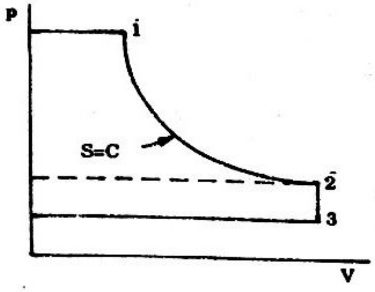
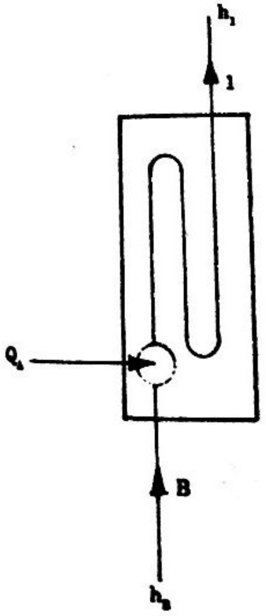


Fig. 3-25. Incomplete-Expansion Engine

$$W = W_{1-2} + W_{2-3}$$

$$W = h_1 - h_2 + v_2 (p_2 - p_3)$$

Heat Added, $Q_A$



$$E_{in} = E_{out}$$

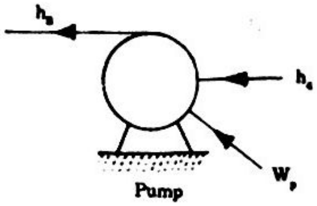
$$Q_A + h_B = h_1$$

$$Q_A = h_1 - h_B$$

Incomplete expansion engine almost always operate with a low initial pressure, hence, the pump work is very small so that it can be neglected.

From the pump energy balance

$$h_B = h_4 + W_p$$



$$W_p \approx 0$$

$$h_B \approx h_4$$

$$Q_A = h_1 - h_4$$

### Thermal Efficiency, $e_c$

$$e_c = \frac{W_{net}}{Q_A} = \frac{W - W_p}{Q_A}$$

$$e_c = \frac{h_1 - h_2 + v_2(p_2 - p_3)}{h_1 - h_4}$$

## The Incomplete-Expansion Engine

### Energy Chargeable, $E_c$

$$E_c = h_1 - h_{f3}$$

### Thermal Efficiency, e

$$e = \frac{W}{E_c} = \frac{h_1 - h_2 + v_2(p_2 - p_3)}{h_1 - h_{f3}}$$

### Mean Effective Pressure, mep or $p_m$

(a) Ideal Mean Effective Pressure

$$p_m = \frac{W}{V_D} = \frac{h_1 - h_2 + v_2(p_2 - p_3)}{v_2}$$

(b) Indicated Mean Effective Pressure, $mep_i$

$$mep_i = \frac{W_i}{V_D}$$

(c) Brake Mean Effective Pressure, $mep_b$

$$mep_b = \frac{W_B}{V_D}$$

(d) Combined Mean Effective Pressure, $mep_k$

$$mep_k = \frac{W_K}{V_D}$$

### Steam Rate, m

(a) Ideal Steam Rate, m

$$m = \frac{3600}{W}$$

(b) Indicated Steam Rate, $m_i$

$$m_i = \frac{3600}{W}$$

(c) Brake Steam Rate, $m_b$

$$m_b = \frac{3600}{W_I}$$

(d) Combined Steam Rate, $m_k$

$$m_k = \frac{3600}{W_K}$$

### Thermal Efficiency, e

(a) Indicated Thermal Efficiency, $e_i$

$$e_i = \frac{W_I}{E_c}$$

(b) Brake Thermal Efficiency, $e_b$

$$e_b = \frac{W_B}{E_c}$$

(c) **Combined Thermal Efficiency, $e_k$**

$$e_k = \frac{W_K}{E_c}$$

**Engine Efficiency, n**

(a) **Indicated Engine Efficiency, $n_i$**

$$n_i = \frac{\text{Indicated Power}}{\text{Ideal Power}} = \frac{\text{Indicated Work}}{\text{Ideal Work}}$$

$$n_i = \frac{W_I}{W}$$

(b) **Brake Engine Efficiency, $n_b$**

$$n_b = \frac{\text{Brake Power}}{\text{Ideal Power}} = \frac{\text{Brake Work}}{\text{Ideal Work}}$$

$$n_b = \frac{W_B}{W}$$

(c) **Combined Engine Efficiency, $n_k$**

$$n_k = \frac{\text{Combined Power}}{\text{Ideal Power}} = \frac{\text{Combined Work}}{\text{Ideal Work}}$$

$$n_k = \frac{W_K}{W}$$

(d) **Mechanical Efficiency, $n_m$**

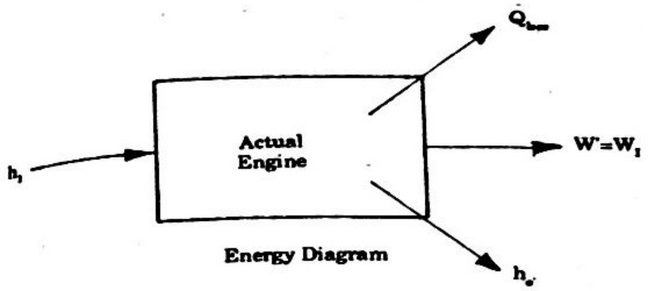
$$n_m = \frac{\text{Brake Power}}{\text{Indicated Power}} = \frac{\text{Brake Work}}{\text{Indicated Work}}$$

$$n_m = \frac{W_B}{W_I}$$

(e) **Generator Efficiency, $n_g$**

$$n_g = \frac{W_K}{W_B}$$

**Approximate Enthalpy of Exhaust Steam, $h_{e'}$,**



Energy Diagram

*where:* $W'$ = actual work
$W' = W_I$, indicated work

Energy balance:

$$h_1 = Q_{loss} + W_I + h_{e'}$$

$$h_{e'} = h_1 - Q_{loss} - W_I$$

**Relationship Between Efficiencies and Steam Rates.**

(1) $e_i = n_i e$

Proof: $e_i = \frac{W_I}{E_c}$

$$e_i = n_i e = \frac{W_I}{W} \bullet \frac{W}{E_c} = \frac{W_I}{E_c}$$

(2) $e_b = n_b e$

Proof: $e_b = \frac{W_B}{E_c}$

$$e_b = n_b e = \frac{W_B}{W} \bullet \frac{W}{E_c} = \frac{W_B}{E_c}$$

(3) $e_k = n_k e$

Proof: $e_k = \frac{W_K}{E_c}$

$$e_k = n_k e = \frac{W_K}{W} \bullet \frac{W}{E_c} = \frac{W_K}{E_c}$$

(4) $e_k = n_g n_m e_i$

Proof: $e_k = n_g n_m e_i$

$$e_k = \frac{W_K}{W_B} \bullet \frac{W_B}{W_I} \bullet \frac{W_I}{E_c} = \frac{W_K}{E_c}$$

(5) $n_m = \frac{m_i}{m_b}$

Proof: $n_m = \frac{W_B}{W_I}$

$$n_m = \frac{m_i}{m_b} = \frac{3600/W_I}{3600/W_B} = \frac{W_B}{W_I}$$

(6) $n_m = \frac{e_b}{e_i}$

Proof: $n_m = \frac{e_b}{e_i} = \frac{W_B/E_c}{W_I/E_c} = \frac{W_B}{W_I}$

(7) $n_g = \frac{e_k}{e_b}$

Proof: $n_g = \frac{e_k}{e_b} = \frac{W_K/E_c}{W_B/E_c} = \frac{W_K}{W_B}$

(8) $n_k = n_i n_m n_g$

Proof: $n_k = \frac{W_K}{W}$

$$n_k = n_i n_m n_g = \frac{W_I}{W} \bullet \frac{W_B}{W_I} \bullet \frac{W_K}{W_B} = \frac{W_K}{W}$$

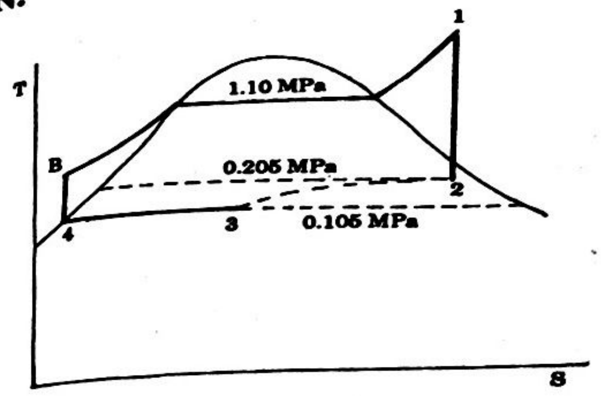
(9) $m_k = \frac{m_i}{n_g n_m}$

Proof: $m_k = \frac{3600}{W_K}$

$$m_k = \frac{m_i}{n_g n_m} = \frac{\frac{3600}{W_I}}{\frac{W_K}{W_B} \bullet \frac{W_B}{W_I}} = \frac{3600}{W_K}$$

(3-19) steam at 1.10 MPa and 205°C is delivered to the throttle of an engine. The steam expands to 0.205 MPa, where release occurs. Exhaust is at 0.105 MPa. A test of the engine showed an indicated steam consumption of 13.28 kg/kwh and a mechanical efficiency of 85%. Find (a) the ideal work and ideal thermal efficiency, (b) the ideal steam rate, (c) the brake and indicated works, (d) the brake thermal efficiency, (e) the brake engine efficiency, (f) the mep of the ideal engine and the indicated mep.

**SOLUTION:**



$p_1 = 1.10$ MPa $\qquad h_1 = 2834.2$ kJ/kg

$t_1 = 205°C \qquad s_1 = 6.6659$ kJ/(kg) (K)

At $p_2 = 0.205$ MPa

$v_{f2} = 0.0010613 \qquad hf2 = 508.03 \qquad s_{f2} = 1.5386$

$v_{fg2} = 0.86444 \qquad h_{fg2} = 2199.8 \qquad s_{fg2} = 5.5803$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.6659 - 1.5386}{5.5803} = 0.9188$$

$h_3 = h_{f2} + x_2 h_{fg2} = 508.03 + (0.9188)(2199.8) = 2529.2$ kJ/kg

$v_3 = v_{f2} + x_2 s_{fg2} = 0.0010613 + (0.9188)(0.86444)$

$= 0.7953$ m³/kg

$h_4 = h_{f3} = h_f$ at 0.105 MPa $= 423.24$ kJ/kg

a) $W = h_1 - h_2 + v_2(p_2 - p_3) = 2834.2 - 2529.2 +$

$(0.7953)(205 - 105)$

$= 384.5$ kJ/kg

$$e_e = \frac{W}{h_1 - h_{f3}} = \frac{384.5}{2834.2 - 423.24} = 0.1595 \text{ or } 15.95\%$$

$$\text{(b) } m = \frac{3600}{W} = \frac{3600}{384.5} = 9.363 \text{ kg/kwh}$$

$$\text{(c) } W_I = \frac{3600}{m_i} = \frac{3600}{13.28} = 271.1 \text{ kJ/kg}$$

$$W_B = (n_m)(W_I) = (0.85)(271.1) = 230.4 \text{ kJ/kg}$$

$$\text{(d) } e_b = \frac{W_B}{h_1 - h_{f3}} = \frac{230.4}{2834.5 - 423.24} = 0.0956 \text{ or } 9.56\%$$

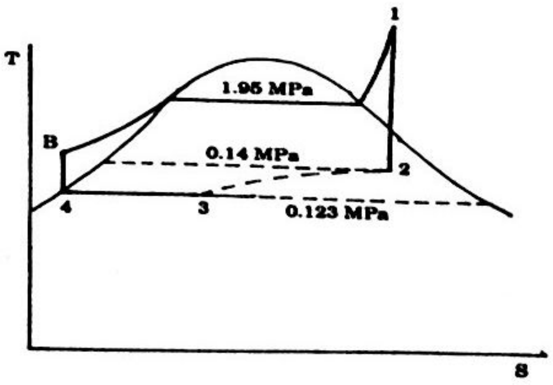
$$\text{(e) } n_b = \frac{W_B}{W} = \frac{230.4}{384.5} = 0.5992 \text{ or } 59.92\%$$

$$\text{(f) Ideal mep} = \frac{W}{v_2} = \frac{384.5}{0.7953} = 483.47 \text{ kPa}$$

$$\text{Indicated mep} = \frac{W_I}{v_2} = \frac{271.1}{0.7953} = 340.88 \text{ kPa}$$

(3-20) An incomplete expansion engine is supplied with steam at 1.95 MPa and 400°C. Release occurs at 0.14 MPa and exhaust at 50°C. Test of the engine showed a brake steam rate of 12.2 kg/kwh and a mechanical efficiency of 82.5%. Find (a) the ideal work and the ideal thermal efficiency, (b) the ideal steam rate, (c) the indicated thermal efficiency, (d) the indicated engine efficiency, (e) the brake mep, and (f) the ideal thermal efficiency of an incomplete expansion cycle operating under the same conditions.

**SOLUTION:**



$p_1$ = 1.95 MPa $\qquad$ $h_1$ = 3248.5 kJ/kg

$t_1$ = 400°C $\qquad$ $s_1$ = 7.1397 kJ/(kg) (K)

At $p_2$ = 0.14 MPa

$v_{f2}$ = 0.001051 $\qquad$ $h_{f2} = 458.39$ $\qquad$ $s_{f2} = 1.4109$

$v_{fg2}$ = 1.23555 $\qquad$ $h_{fg2} = 2232.1$ $\qquad$ $s_{fg2} = 5.8355$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{7.1397 - 1.4109}{5.8355} = 0.9817$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 458.39 + (0.9817)(2232.1) =$$

$$= 2649.6 \text{ kJ/kg}$$

$$v_2 = v_{f2} + x_2 v_{fg2} = 0.001051 + (0.9817)(1.23555)$$

$$= 1.214 \text{ m}^3\text{/kg}$$

$$h_4 = h_{f3} = h_f \text{ at } 50°\text{C} = 209.33 \text{ kJ/kg}$$

$$p_3 = \text{saturation pressure at } 50°\text{C} = 0.012349 \text{ MPa}$$

$$W_p = v_{f4}(p_1 - p_4) = (0.0010121)(1950 - 12.349)$$

$$= 1.96 \text{ kJ/kg}$$

$$h_B = h_4 + W_p = 209.33 + 1.96 = 211.3 \text{ kJ/kg}$$

(a) $$W = h_1 - h_2 + v_2(p_2 - p_3) = 3248.5 - 2649.6 + (1.214)(140 - 12.349) = 753.9 \text{ kJ/kg}$$

$$e_e = \frac{W}{h_1 - h_{f3}} = \frac{753.9}{3248.5 - 209.33} = 0.2481 \text{ or } 24.81\%$$

(b) $$m = \frac{3600}{W} = \frac{3600}{753.9} = 4.775 \text{ kg/kwh}$$

(c) $$W_B = \frac{3600}{m_b} = \frac{3600}{12.2} = 295.1 \text{ kJ/kg}$$

$$W_I = \frac{W_B}{n_m} = \frac{295.1}{0.825} = 357.7 \text{ kJ/kg}$$

$$e_i = \frac{W_I}{h_1 - h_{f3}} = \frac{357.7}{3248.5 - 209.33} = 0.1177 \text{ or } 11.77\%$$

(d) $$n_i = \frac{W_I}{W} = \frac{357.7}{753.9} = 0.4745 \text{ or } 47.45\%$$

(e) $$p_{mB} = \frac{W_B}{v_2} = \frac{295.1}{1.214} = 243.1 \text{ kPa}$$

(f) $$W_{net} = W - W_p = 753.9 - 1.96 = 751.9 \text{ kJ/kg}$$

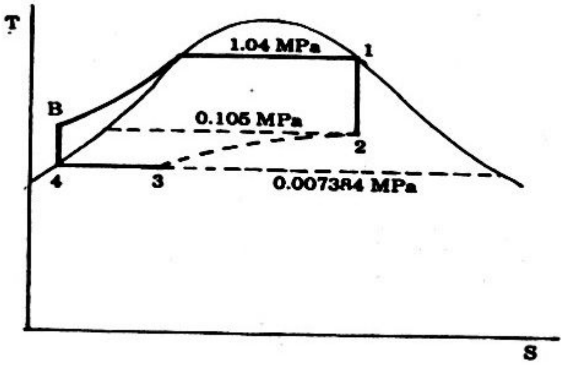
$$Q_A = h_1 = h_B = 3248.5 - 211.3 = 3037.2 \text{ kJ/kg}$$

$$e_c = \frac{W_{net}}{Q_A} = \frac{751.9}{3037.2} = 0.2476 \text{ or } 24.76\%$$

(3-21) Steam engine with incomplete expansion receives saturated steam at 1.04 MPa, expands it to 0.105 MPa where release occurs. Exhaust is to a condenser at 40°C. The actual engine uses 6070 kg/h of steam; it has a brake engine efficiency of 60% and a mechanical efficiency of 85%. The electric

generator which it drives has an efficiency of 92%. (a) For the ideal engine, find the thermal efficiency and the mep. For the actual engine, determine (b) the brake work in kW and the brake thermal efficiency (c) the indicated work in kW, indicated thermal efficiency and indicated steam rate, (d) the combined thermal efficiency and the combined steam rate.

**SOLUTION:**



$$p_1 = 1.04 \text{ MPa, saturated}$$

$$h_1 = h_g \text{ at } 1.04 \text{ MPa} = 2779.6 \text{ kJ/kg}$$

$$s_1 = s_g \text{ at } 1.04 \text{ MPa} = 6.5729 \text{ kJ/(kg)(K)}$$

At $p_2 = 0.105$ MPa

$$v_{f2} = 0.0010443 \qquad h_{f2} = 423.24 \qquad s_{f2} = 1.3181$$

$$v_{fg2} = 1.6174 \qquad h_{fg2} = 2254.4 \qquad s_{fg2} = 6.0249$$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.5729 - 1.3181}{6.0249} = 0.8722$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 423.24 + (0.8722)(2254.4) = 2389.5 \text{ kJ/kg}$$

$$v_2 = v_{f2} + x_2 v_{fg2} = 0.0010443 + (0.8722)(1.6174)$$

$$= 1.4117 \text{ m}^3\text{/kg}$$

$h_{f3} = h_f$ at 40°C = 167.57 kJ/kg

$p_3 =$ saturation pressure at 40°C = 0.007384 MPa

(a) $W = h_1 - h_2 + v_2\,(p_2 - p_3) = 2779.6 - 2389.5$

$+ (1.4117)\,(105 - 7.384) = 527.9$ kJ/kg

$$e = \frac{W}{h_1 - h_{f3}} = \frac{527.9}{2779.6 - 167.57} = 0.2021 \text{ or } 20.21\%$$

$$\text{mep} = \frac{W}{v_2} = \frac{527.9}{1.4117} = 373.9 \text{ kPa}$$

(b) $W_B = (n_b)\,(W) = (0.60)\,(527.9) = 316.7$kJ/kg

$$= \frac{(316.7)\,(6070)}{3600} = 534.\text{kW}$$

$$e_b = \frac{W_B}{h_1 - h_{f3}} = \frac{316.7}{2779.6 - 167.57} = 0.1212 \text{ or } 12.12\%$$

(c) $W_I = \dfrac{W_B}{n_m} = \dfrac{316.7}{0.85} = 372.6$ kJ/kg

$$W_I = \frac{534}{0.85} = 628.2 \text{ kW}$$

$$e_i = \frac{W_I}{h_1 - h_{f3}} = \frac{372.6}{2779.6 - 167.57} = 0.1426 \text{ or } 14.26\%$$

(d) $W_K = (W_B)$ (generator efficiency) = (316.7) (0.92)

= 291.4 kJ/kg

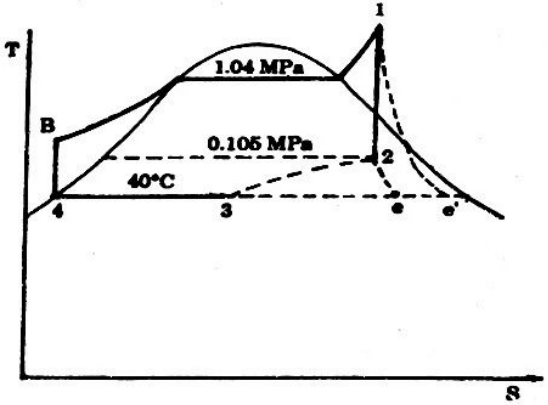
$$e_k = \frac{W_B}{h_1 - h_{f3}} = \frac{291.4}{2779.6 - 167.57} = 0.1116 \text{ or } 11.16\%$$

$$m_k = \frac{3600}{W_K} = \frac{3600}{291.4} = 12.35 \text{ kg/kwh}$$

---

**(3-22)** **A compound steam engine with incomplete expansion receives saturated steam at 1.04 MPa, expands it to 0.105 MPa where release occurs. Exhaust is to a condenser of 40°C. The actual engine uses 1252 kg/hr of steam; it has a brake engine efficiency at 60% and a mechanical efficiency of 85%. The electric generator which it drives has an efficiency of 92%. (a) For the ideal engine, find the thermal efficiency and mep. For actual engine, determine (b) the break work in kJ/kg and kW output and the break thermal efficiency (c) the indicated work, thermal efficiency and steam rate, (d) combined thermal efficiency and steam rate. (e) What is the approximate enthalpy of the actual exhaust steam?**

**SOLUTION:**



$p_1$ = 1.04 MPa and saturated $h_1 = h_g$ at 1.04 MPa

= 2779.6 kJ/kg

$s_1 = s_g$ at 1.04 MPa = 6.5729 kJ/(kg) (K)

At $p_2$ = 0.105 MPa

$v_{f2} = 0.0010443$ $\quad h_{f2} = 423.24$ $\quad s_{f2} = 1.3181$

$v_{fg2} = 1.6184$ $\quad h_{fg2} = 2254.4$ $\quad s_{fg2} = 6.0249$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.5729 - 1.3181}{6.0249} = 0.8722$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 423.24 + (0.8722)(2254.4) = 2389.5 \text{ kJ/kg}$$

$$v_2 = v_{f2} + x_2 v_{fg2} = 0.0010433 + (0.8722)(1.6184)$$

$$= 1.4126 \text{ m}^3\text{/kg}$$

$$h_4 = h_{f3} = h_f \text{ at } 40°\text{C} = 167.57 \text{ kJ/kg}$$

$$\text{(a) } W = h_1 - h_2 + (v_2 (p_2 - p_3) = 2779.6 - 2389.5 + (1.4126)$$

$$(105 - 7.384) = 528 \text{ kJ/kg}$$

$$e = \frac{W}{h_1 - h_{f3}} = \frac{528}{2779.6 - 167.57} = 0.2021 \text{ or } 20.21\%$$

$$\text{mep} = \frac{W}{v_2} = \frac{528}{1.4126} = 373.8 \text{ kN/m}^2$$

$$\text{(b) } W_B = n_b (W) = 0.60 (528) = 316.8 \text{ kJ/kg}$$

$$\text{kW output} = (m)\left(\frac{W_B}{3600}\right) = 1252\left(\frac{316.8}{3600}\right) = 110.2 \text{ kW}$$

$$e_b = \frac{W_B}{h_1 - h_{f3}} = \frac{316.8}{2779.6 - 167.57} = 0.1213 \text{ or } 12.13\%$$

$$\text{(c) } W_I = \frac{W_B}{n_m} = \frac{316.8}{0.85} = 372.7 \text{ kJ/kg}$$

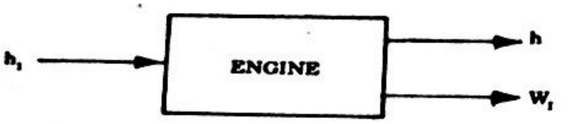
$$e_i = \frac{W_I}{h_1 - h_{f3}} = \frac{372.7}{2779.6 - 167.57} = 0.1427 \text{ or } 14.27\%$$

$$m_i = \frac{3600}{W_I} = \frac{3600}{372.7} = 9.659 \text{ kg/kwh}$$

$$\text{(d) } e_k = (e_B)(n_g) = (12.13)(0.92) = 11.16\%$$

$$m_k = \frac{3600}{(W_B)(n_g)} = \frac{3600}{(316.8)(0.92)} = 12.35 \text{ kg/kwh}$$

(e)



$$h_{e'} = h_1 - W_I = 2779.6 - 372.7 = 2406.9 \text{ kJ/kg}$$

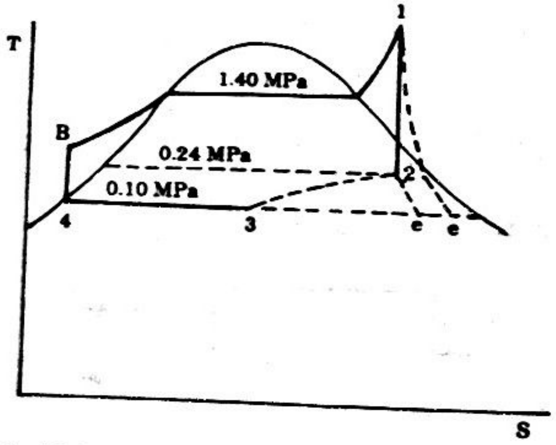
$$h_{e'} = h_{f3} + x_{e'} h_{fg3} = 167.57 + (x_{e'})(2406.7)$$

$$x_{e'} = \frac{h_{e'} - h_{f3}}{h_{fg3}} = \frac{2406.9 - 167.57}{2406.7} = 0.9304 \text{ or } 93.04\%$$

(3-24) A 200 kw steam engine has the following data:
Throttle: 1.40 MPa, 205°C
Steam flow: 3020 kg/hr
Release pressure: 0.24 MPa
Exhaust pressure: 0.10 MPa
Mechanical efficiency: 83%

Determine (a) the enthalpy of the exhaust from the ideal engine, (b) the enthalpy and quality of the exhaust from the actual engine, and (c) the lost work of actual engine.

**SOLUTION:**



$p_1 = 1.40$ MPa $\qquad h_1 = 2816.6$ kJ/kg

$t_1 = 205$°C $\qquad s_1 = 6.5254$ kJ/(kg) (K)

At $p_2 = 0.24$ MPa

$v_{f2} = 0.001066$ $h_{f2} = 529.65$ $s_{f2} = 1.593$

$v_{fg2} = 0.7467$ $h_{fg2} = 2185.4$ $s_{fg2} = 5.4733$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.5254 - 1.593}{5.4733} = 0.9012$$

$h_2 = h_{f2} + x_2 h_{fg2} = 529.65 + (0.9012)(2185.4) = 2499.1$ kJ/kg

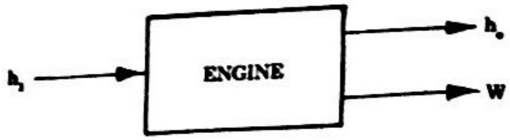
$v_2 = v_{f2} + x_2 v_{fg2} = 0.001066 + (0.9012)(0.7467)$

$= 0.6721$ m³/kg

$h_4 = h_{f3} = h_f$ at 0.10 MPa $= 417.46$ kJ/kg

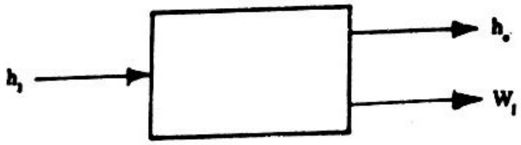
(a) $W = h_1 - h_2 + v_2(p_2 - p_3) = 2816.6 - 2499.1 + (0.6721)(240 - 100) = 411.6$ kJ/kg

For the ideal engine, from the first law of Thermodynamics, $E_{in} = E_{out}$



$$h_e = h_1 - W = 2816.6 - 411.6 = 2405 \text{ kJ/kg}$$

(b) For the actual engine



$$m_b = \frac{m}{W_B} = \frac{3020 \text{ kg/hr}}{200 \text{ kw}} = 15.1 \text{ kg/kwh}$$

$m_i = (n_m)(m_b) = (0.83)(15.1) = 12.53$ kg/kwh

$$W_I = \frac{3600}{m_i} = \frac{3600}{12.53} = 287.3 \text{ kJ/kg}$$

$h_{e'} = h_1 - W_I = 2816.6 - 287.3$ kJ/kg $= 2529\text{-}3$ kJ/kg

At 0.10 MPa; $h_f = 417.46$ kJ/kg and $h_{fg} = 2258$ kJ/kg

$h_{e'} = h_f + x_e h_{fg}$

$2529.3 = 417.46 + (x_{e'})(2258)$

$$x_{e'} = \frac{2529.3 - 417.46}{2258} = 0.9353 \text{ or } 93.53\%$$

(c) Lost Work $= W - W_I = 411.6 - 287.3 = 124.3$ kJ/kg

## Superposition or Topping

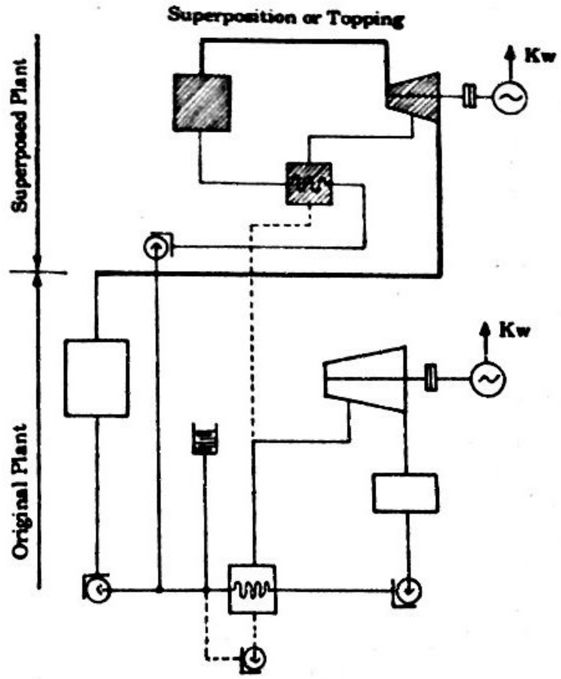


**Fig. 3-26. High-pressure Unit Superposed on Existing Plant.**

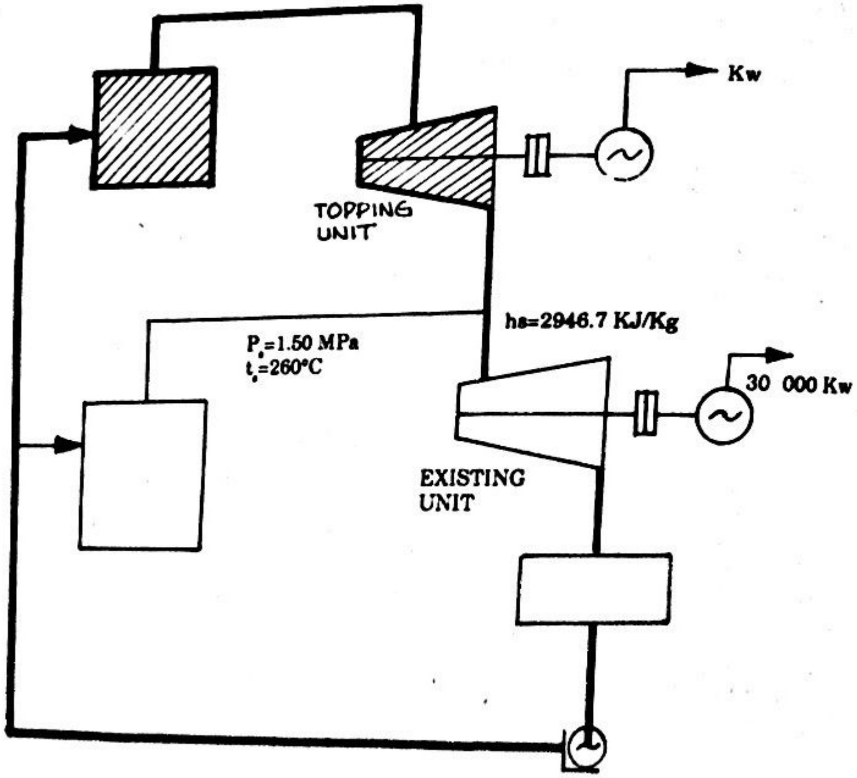
Superposition or topping unit is a new set of high-pressure equipment to be added or topped into the existing system with the idea of increasing the capacity of the whole system and at the same time replacing the old boiler (oil-fired) with a new high-pressure steam generator (coal-fired). As shown in the diagram the new Hp turbine or the topping unit is a non-condensing turbine and its exhaust will be utilized by the old low-pressure turbine.

(3-24) a 30,000 kW existing plant has the following throttle conditions:

$$P_s = 1.50\text{ MPa} \qquad t_s = 260°\text{C}$$

The steam rate of this plant is 5.67 Kg/kwh. An additional 12,500 kw is wanted from this superposed unit using an average indicated engine efficiency of 78% and a mechanical-electrical efficiency of 96%, estimate the steam conditions of the superposed plant.

**SOLUTION:**



$$h_s = h \text{ at } 1.5\text{ MPa, } 260°\text{C} = 2946.7\text{ kJ/kg}$$

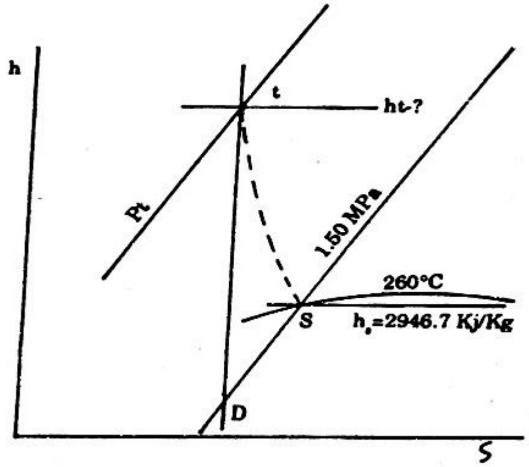
$$m_s = (m_k)(P_k) = (5.67)(30{,}000) = 170{,}100\text{ kg/hg}$$

For the topping unit:

$$m_{k'} = \frac{m_s}{\text{output}} = \frac{170100}{12500} = 13.6\text{ kg/kwh}$$

$$W_k = \frac{3600}{m_{k'}} = \frac{3600}{13.6} = 264.7\text{ kJ/kg}$$

$$W_I = \frac{W}{n_{me}} = \frac{264.7}{0.96} = 275.73\text{ kJ/kg}$$



$$W_I = h_t - h_s$$

$$275.73 = h_t - 2946.7$$

$$h_t = 3222.43\text{ kJ}$$

$$n_I = \frac{h_t - h_s}{h_t - h_o} = 0.78$$

$$\frac{W_I}{h_t - h_o} = \frac{275.73}{3222.43 - h_o} = 0.78$$

$$h_o = 3222.43 - \frac{(275.73)}{(0.78)} = 2868.93 \text{ kJ/kg}$$

At point 0, $p_o = 1.5$ MPa

$h_o = 2868.9$ kJ/kg

then $s_o = 6.52$ kJ/kg

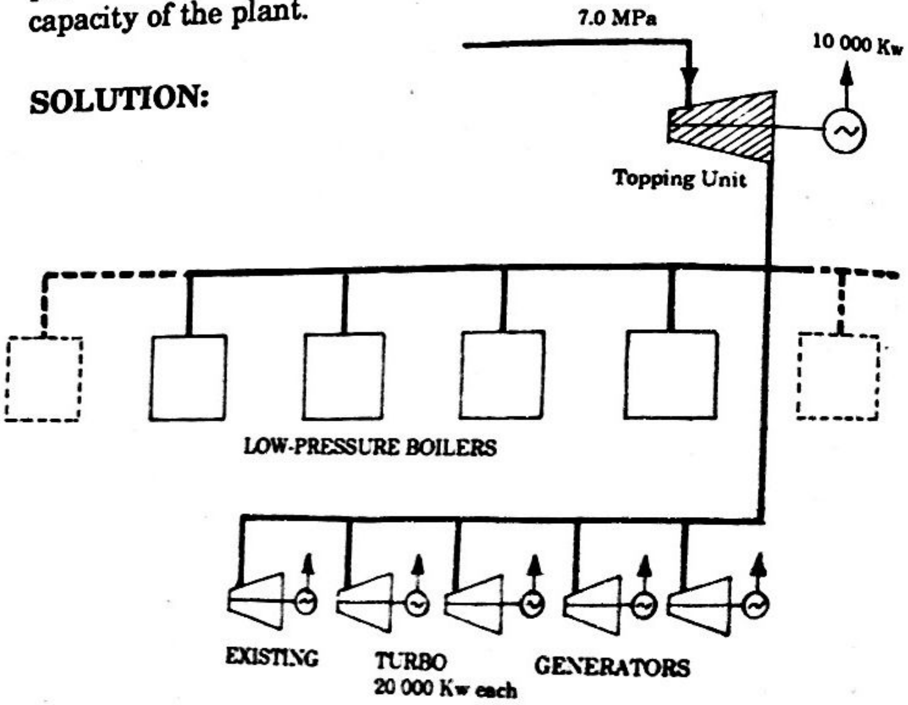
At point t, $st = s_o = 6.52$ kJ/kg

$h_t = 3222.43$ kJ/kg

Then pt = 6.0 MPa and $t_t = 416°C$

(3-25) A plant having five 20,000 kW units using steam at 1.5 MPa and 260°C has a full-load steam rate of 5.9 kg/kwh. The steaming capacity of the existing 10 boilers exactly matches the turbine requirements. If this plant were topped by a 10,000 kW turbine having an engine efficiency of 77.6% (indicated), mechanical efficiency of 99% and a generator efficiency of 98%, determine (a) the initial steam temperature required for 7 MPa steam pressure, (b) the steam exhausted from the topping turbine, and (c) the number of existing low-pressure boilers that must be retained to develop the full capacity of the plant.

**SOLUTION:**


7.0 MPa
10 000 Kw
Topping Unit
LOW-PRESSURE BOILERS
EXISTING
TURBO
20 000 Kw each
GENERATORS

**m = $(m_k)$ (kw output) = (5.9) (20,000) = 118000 kg/hr**

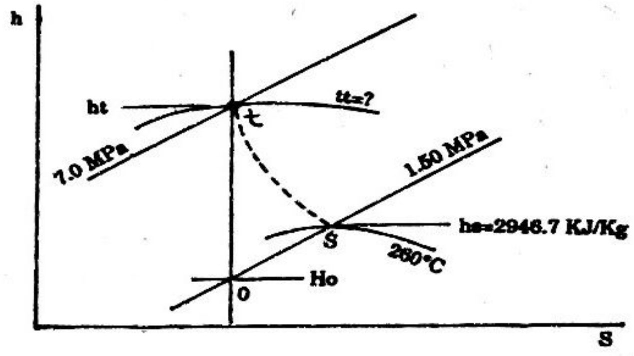
**Total steam flow:**

$$
\begin{aligned}
m_T &= \text{(no. of boilers) (m)}\\
&= (5)(118000)\\
&= 590{,}000 \text{ kg/hr}
\end{aligned}
$$

**For the existing units:**

**$h_s$ = h at 1.5 MPa, 260°C = 2946.7 kJ/kg**

**On Miller's Chart by cut and try method:**


h
ht
tt=?
t
7.0 MPa
1.50 MPa
hs=2946.7 KJ/Kg
S
260°C
Ho
0
S

Try $t_t$ = 435°C

$h_t$ = h at 7.0 MPa, 435°C = 3250 kJ/kg

$h_o$ = h at 1.50 MPa and $s_o$ equal to $s_t$ = 2860 kJ/kg

$$\text{(a) } n_i = \frac{h_t - h_s}{h_t - h_o}$$

subsituting the values

$$0.776 - \text{approximately equal to} - \frac{3250 - 2946.7}{3250 - 2860}$$

Therefore: $t_t$ = 435°C

(b) Exhaust steam from the topping unit, $m_t$

$$W_l = h_t - h_e = 3250 - 2946.7 = 303.3 \text{ kJ/kg}$$

$$W_k = (n_m)(n_g)(W_l) = (0.99)(0.98)(303.3) = 294.26 \text{ kJ/kg}$$

$$m_t = \frac{\text{kw output}}{W_k} = \frac{10{,}000}{294.26} = 34 \text{ kg/sec or } 122{,}400 \text{ kg/hr}$$

(c) With 10 boilers generating a total of 590,000 kg/hr, then each boiler has to generate 59,000 kg/hr. The new boiler is generating 122400 kg/h for the topping turbine.

Therefore, the number of boilers to be retained

$$= \frac{590{,}000 - 122{,}400}{59{,}000}$$

$$= 7.92 \text{ or retain 8 boilers}$$

## Problems

### Rankine Cycle and Engine

1. In an ideal Rankine cycle, steam is generated at 4.1 MPa and 480°C. The condenser is at 32°C. Determine (a) the ideal pump work, (b) the cycle thermal efficiency, (c) For an engine with the same end states, determine its thermal efficiency, steam rate, and mep. (d) Considering the engine only, assume the brake engine efficiency to be 78%, the generator efficiency 92%, the steam flow rate 163,600 kg/hg; compute the output of the combined unit. Ans. (a) 4.12 kJ/kg; (b) 38.66%; (c) 38.73%, 2.85 kg/kwh, 51.92 kPa; (d) 41,227 kW

2. A Rankine cycle has the following operating conditions:

Steam pressure, 2.5 MPa
Steam temperature, 280°C
Exhaust pressure, 0.026 MPa
Indicated steam rate, 5.45 kg/kwh
Mechanical efficiency, 90%

For ideal cycle and engine, calculate (a) the thermal efficiency, (b) the engine steam rate), (c) the actual condition of exhaust steam, and (d) the steam flow rate in kg/h for 1 Mw generator output at 94% efficiency. Ans. (a) 28.6%, 28.5%; (b) 4.69 kg/kwh; (c) 86.33%; (d) 6447 kg/h

3. A Rankine turbo-generator unit receives 27,000 kg/h of steam at 1.2 MPa and superheat by 122 degrees. Condensate leaves at 22°C. For a brake steam rate of 4.74 kg/kwh, mechanical efficiency of 90% and generator efficiency of 94%, determine (a) the ideal cycle thermal efficiency, (b) the indicated engine efficiency, (c) the condition of turbine actual exhaust, and (d) the turbo-generator output in Kw. Ans. (a) 33%, (b) 85.7%, (c) 87%, (d) 5354.5 kw

4. The condensing pressure for a Rankine engine is 1 bar. Calculate the net work for 1 kg of steam and the thermal efficiency when the steam at the beginning of expansion is at 50 bar and (a) saturated, (b) 350°C, and (c) 640°C.

Ans. (a) 627.2 kJ/kg, 26.4%; (b) 727.3 kJ/kg, 27.5%; (c) 1079.49 kJ/kg, 32.35%

5. There are received 90,000 kg/h of steam at 2.15 MPa, 480°C by a Rankine engine; exhaust occurs at 0.10 MPa. For the engine find (a) W, (b) m, and (c) $e_e$. For the cycle find (a)

$W_{net}$, and (b) $e_c$. Ans. (a) 18,767.5 kW, (b) 4.8 kg/kwh, (c) 24.98%, (d) 748.56 kJ/kg, (e) 24.93%

6. A turbo-generator has a combined steam rate of 5.35 kg/kwh at its rated output of 20,000 kW. The steam is supplied at 17 bar, 300°C and exhaust at 0.1 bar. Calculate (a) the combined heat rate, (b) the combined thermal efficiency, (c) the engine efficiency, and (d) the combined work. Ans. (a) 15,195.6 kJ/kg, (b) 23.7%, (c) 672.9 kJ/kg, (d) 78.1%

## Reheat Cycle andEngine

7. A reheat cycle with two stages of reheating is executed, with steam expanding initially from 90 bar and 530°C. The two reheater pressures are 10 bar and 0.6 bar, and the steam leaves each reheater at 480°C. Condensation occurs at 0.03 bar. For an ideal cycle, find (a) $Q_A$ and e. (b) For the engine, ignore the pressure drop through the reheaters, let the engine operate through the same states, and compute W and e. (c) What is the steam flow rate for an engine output of 20,000 kW? Ans. (a) 4657.5 kJ/kg, 44.31%; (b) 2072.6 kJ/kg, 44.41%; (c) 9.65 kg/s

8. An ideal reheat turbine with one stage of reheat receives steam at 8.0 MPa, 480°C. Reheat pressure is 0.83 MPa and exhaust pressure is 0.007 MPa. The work is 1559 kJ/kg and the thermal efficiency is 41.05%. Find the temperature of steam leaving the reheater. Ans. 455°C

9. In a reheat cycle steam at 15 MPa, 540°C enters the engine and expands to 1.95 MPa. At this point the steam is withdrawn and passed through a reheater. It reenters the engine at 540°C. Expansion now occurs to the condenser pressure of 0.0035 MPa. (a) For the ideal cycle, find $e_e$ (b) A 60,000 kw turbine operates between the same state points except that the steam enters the reheater at 1.95 MPa and 260°C, departs at 1.8 MPa and 540°C. The steam flow is 147,000 kg/hr; generator efficiency is 96%. For actual engine, find $e_k$, $m_k$, and $n_k$, (c) Determine the approximate enthalpy of the exhaust steam if the heat lost through the turbine casing is 2% of the combined work.

Ans.: (a) 46.1%, (b) 37.3%, 2.45 kg/kwh, 79.3%, (c) 2489.6 kJ/kg

10. Steam at 200 bar, 760°C enters the throttle of a reheater engine and expands to 10 bar. At this point it leaves the engine, enters the reheater, and returns at 9 bar, 600°C, expansion then occurs to the condenser pressure of 0.05 bar ($h_f$ = 138 kJ/kg) on the basis of flow of 1 kg steam. Find (a) W and e of the cycle (b) W, e and m for the ideal engine (c) In the actual engine the steam enters the reheater at 10 bar, 300°C and later expands to a saturated state at the exhaust to the condenser, find W, e and m.

Ans.: (a) 2226.7 kJ/kg, 48.95%; (b) 2246.7 kJ/kg, 49.17%, 1.6 kg; (c) 2052.1 kJ/kg, 45.85%, 1.75 kg/kwh.

## Regenerative Cycle and Engine

11. There are received 68,000 kg/h of steam by an ideal regenerative engine, having only one heater, of which the heater receives 15,400 kg/h; the condenser receives the remainder at 0.007 MPa. If the heater pressure is 0.96 MPa, find the state (quality or °SH) of the steam (a) at the heater entrance and (b) at the condenser entrance.

Ans. (a) saturated, (b) 78.5% quality

12. A turbine with one extraction for regenerative feedwater heating, receives steam with an enthalpy of 3373 kJ/kg and discharges it with an exhaust enthalpy of 2326 kJ/kg. The ideal regenerative feedwater heater receives 11,338 kg/h of extracted steam at 345 kPa (whose h = 2745 kJ/kg). The feedwater (condensate from the condenser) enters the heater with an enthalpy of 140 kJ/kg and departs saturated at 345 kPa ($h_f$ = 582 kJ/kg). Calculate the turbine output in kW.

Ans. 18,116 kW

13. A 10,000 kW turbo-generator operating at rated capacity receives 52,000 kg/h of steam at 2.90 MPa and 310°C, exhaust is aat 0.007 MPa. At 1.20 MPa and 230°C, there are actually extracted 20% of throttle flow for regenerative heating of feedwater. The generator efficiency is 95%. For an ideal engine, find (a) the quantity of steam extracted, kg/h (b) W and (c) e. For the actual engine, find (d) the temperature of feedwater from the open heater, (e) $n_k$ and (f) the approximate enthalpy of exhaust steam and its quality. Ans. (a) 12,428 kg/h; (b) 787.3 kJ/kg, (c) 35.48%, (d) 167.5°C, (e) 87.9%, (f) 2143.8 kJ/kg, 82.2%

14. A 25,000 kW turbo-generator is supplied with 128,000 kg/h of steam at 2.50 MPa and 400°C when developing it rated load. There are actually extracted 10,400 kg/h at 0.3 MPa and

8300 kg/h at 0.06 MPa. The condenser pressure is 0.007 MPa and actual feedwater temperature is 127°C. For the ideal engine find (a) various percentages of extracted steam and (b) e. For the actual engine, find (c) the various percentages of extracted steam, (d) $m_k$, (e) $e_k$, and (f) $n_k$. (g) Find the approximate enthalpy of the exhaust if the generator efficiency if 96%. Ans. (a) 8.5%, 7,8%; (b) 37.1%, (c) 8.12%, 6.48%, (d) 5.12, (e) 26%, (f) 70.7%, (g) 2460 kJ/kg

## Reheat-Regenerative Cycle and Engine

15. A reheat-regenerative engine receives steam at 207 bar and 593°C, expanding it to 38.6 bar, 343°C. At this point, the steam passes through a reheater and reenters the turbine at 34.5 bar, 593°C, hence expands to 9 bar, 492°C, at which point the steam is bled for feedwater heating. Exhaust occurs at 0.07 bar. Beginning at the throttle (point 1), these enthalpies are known (kJ/kg):

$h_1 = 3511.3$ $h_{2'} = 3082.1$

$h_2 = 3010.0$ $h_4 = 3205.4$

$h_3 = 3662.5$ $h_6 = 163.4$

$h_{4'} = 322.9$ $h_{7'} = 723.59$

$h_5 = 2308.1$ $h_7 = 742.83$

For ideal engine, sketch the events on the Ts plane and for 1 kg of throttle steam, find (a) the mass of bled steam, (b) the work, (c) the efficiency, and (d) the steam rate. In the actual case, water enters the boiler at 171°C and the brake engine efficiency is 75% (e) determine the brake work and the brake thermal efficiency. (f) Let the pump efficiency be 65%, estimate the enthalpy of the exhaust steam.

Ans. (a) 0.19 kg/kg, (b) 1685.1 kj/kg, (c) 49.26%, (d) 2.14 kg/kwh, (e) 1263.8 kJ/kg, 37.5%%; (f) 2742 kJ/kg

16. In a 35,000 kW turbo-generator that receives steam at 6.9 MPa and 370°C, 11% of the throttle steam is actually extracted at 2 MPa, 215°C; with the remainder being reheated to 1.8 MPa and 315°C; then 20% of the throttle steam is actually extracted at0.724 MPa, each extraction serving an open feedwater heater. The engine exhaust to a condenser pressure of 0.005 MPa and the temperature of the feedwater from the last heater is 205°C. The combined steam rate of the turbo-generator unitn is 4.898 kg/kwh and the generator efficiency is 95%. For the total throttle flow to an ideal engine, find (a) extracted steam for the last heater, (b) W, (c) e. For the actual engine, find (d) $e_k$, (e) $n_k$. (f) What is the enthalpy of the actual exhaust when the pump efficiency is 60%? Ans. (a) 16,803 kg/h; (b) 1018 kJ/kg; (c) 42%; (d) 30%; (e) 72.2%; (f) 2550 kJ/kg

17. There are developed 25,000 kW by a reheat-regenerative engine (turbogenerator) which receives steam at 4.2 MPa, 205°C and exhaust at 0.007 MPa. At 1.90 MPa and 215‡C, part of the steam is extracted for feedwater heating and the remainder is withdrawn for reheating. The reheated steam enters the turbine at 1.8 MPa and 270°C and expands to 1.38 MPa, where more steam is extracted for feedwater heating and the remainder expands to the condenser pressure of 0.007 MPa and an actual quality of 90%. Feedwater leaves the last heater at a temperature of 207°C. The generator has an efficiency of 95%. For the ideal engine, find (a) the percentages of the extracted steam, (b) W, and (c) e. Let the actual extracted steam be 85% of those for the ideal engine and for the actual engine, find (d) the total throttle flow, if the break work equal the fluid work, (e) $e_k$, and (f) $n_k$.

Ans. (a) 0.035, 0.234; (b) 833 kJ/kg; (c) 37.2% (e) $e_k$, and (f) $n_k$. (d) 147,000 kg/h; (e) 27.5%; (f) 73.6%

## Incomplete-Expansion Cycle and Engine

18. A steam engine receives saturated steam at 260°C. Steam is released when its specific enthalpy is 2200 kJ/kg. Condensate leaves at 0°C. For ideal engine find, (a) the work and the thermal efficiency and (b) mep.

Ans. (a) 680.6 kJ/kg, 27.64%; (b) 631.5 kPa

19. An ideal incomplete-expansion cycle uses steam generated at 1.85 MPa. Release condition is dry and saturated. Exhaust steam is at 0.03 MPa and has 10.58% quality. The actual engine uses 1000 kg/h nof steam. The brake engine efficiency is 60% and the mechanical efficiency is 85%.

(a) For the ideal engine, calculate the thermal efficiency and mep.

For the actual engine, calculate

(b) the brake power in kW and the brake thermal efficiency,

(c) the indicated thermal efficiency and the indicated heat rate,

(d) Compute the approximate enthalpy of the exhaust steam at 0.03 MPa assuming that the heat loss from engine is 23 kJ per kg of steam.

Ans. (a) 19.6%, 1002 kPa/ (b) 92.5 kW, (c) 13.85%, 25,990 kJ/kwh/ (d) 2761.9 kJ/kg

20. An ideal incomplete-expansion cycle uses saturated steam generated at 0.7 MPa. The exhaust pressure is 0.11 MPa, and the release is at 118°C. On the basis of 1 kg., find (a) $Q_A$, (b) $Q_R$, (c) W, (d) the quality of exhaust steam, and (e) the quality of exhaust steam, and (e) the thermal efficiency. An engine with incomplete expansion operates under this conditions. (f) If the indicated engine efficiency is 72% and uses 460 kg per hour of steam, what is the indicated power output in kw? (g) If the mechanical efficiency is 88%, what are the thermal efficiency and the brake thermal efficiency and the brake steam rate?

Ans. (a) 2333.1 kJ/kg, (b) 2034.71 kJ/kg, (c) 298.4 kJ/kg (d) 90.4%, (e) 12.76%, (f) 27.4 kW, (g) 8.1%, 19 kg/kw.hr

21. An engine with incomplete expansion is supplied with steam at 2.75 MPa and 260°C. Release Occurs at 0.35 MPa and the condenser temperature is 42°C. (a) If the brake engine efficiency is 60% and if 1225 kg/hr of steam are used, what is the brake power in kW? (b) If the mechanical efficiency is 87%, what is the indicated thermal efficiency, the indicated work in kJ/kg, and the indicated steam rate? (c) If the engine drives a generator whose efficiency is 92%, what are the combined thermal efficiency and the combined steam rate?

Ans. (a) 111.7 kW, (b) 37[illegible].54 kJ/kg, 13.87, 9.53 kg/kW-hr (c) 11%, 11.9 kg/kW-hr

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## Topping

22. The capacity of a power plant is to be increased by superposing a high-pressure addition on the existing 1.85 MPa plant. Existing turbines are rated to take 370,000 kg/h at 290°C. Assume average indicated engine efficiency of superposed turbine will be 76%, mechanical-electrical efficiency, 93%. What initial steam conditions would suffice to add 20,000 kW to the plant capacity/ Ans. 7.2 MPa, 448°C

23. An existing plant receiving 364,000 kg/h of 1.50 MPa and 250°C steam is to be superposed with a high-pressure turbine to receive steam at 11.0 MPa and 510°C. Nine percent of the topping turbine flow is extracted at 3.0 MPa and 340°C. For a mechanical-electrical efficiency of 93%, determine the kW output of the topping unit. Ans. 46,391 kW.

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